

Rational Expression: any algebraic expression that can be written as the quotient of 2 polynomials.

X-terms in numerator + denom.

Miscellaneous Topics - Rational Expressions

★ Simplifying Expressions → What can you do?

→ If both the numerator and denominator have ONLY multiplication you can create a **prime factor tree**.

$$\frac{20}{33} \times \frac{11}{50}$$

(1) Split each # into its prime factors.

$$\frac{(2^2)(5)}{(3)(11)} \times \frac{(11)}{(5)^2(2)}$$

(2) Multiply the 2 fractions

$$\frac{(2^2)(5)(11)}{(3)(11)(5^2)(2)}$$

(3) look for same bases and begin to use exponent laws to cancel.

$$\frac{(2) \cancel{(5)} \cancel{(11)}}{(3) \cancel{(11)} \cancel{(5^2)} \cancel{(2)}} \rightarrow \frac{2}{(3)(5)} = \frac{2}{15}$$

Prime factor tree for 20: $20 \swarrow 2$

smallest prime
x what = 20

10 can
be broken
up into smallest
prime # x what?

$$10 \swarrow 2$$

prime # so
tree ends here.

After drawing the tree, count how many prime factors there were and multiply all your prime factors by the last prime number.

$$SO: 2 \times 2 \times 5 = (2^2)(5)$$

→ If your numerator/denominator has addition or subtraction, DON'T do prime factorization! Instead factor by:

- difference of squares
- GCF
- criss cross
- product and sum
- quadratic formula

(a) Stating domain restrictions when simplifying rational expressions

You need to state domain restrictions with rational expressions because restrictions present/show holes in the graph.

Domain restrictions include: denominator $\neq 0$ radicand ≥ 0

Look for restrictions:

- before/after flip when dividing
- at your simplified/factored denominator

When simplifying rational expressions, binomials that remain/stay create vertical asymptotes. Binomials that cancel create holes (in the graph.)

Simplify the following:

$$\frac{x^2 - 49}{2x^2 + 2x - 84}$$

difference of squares

$$= \frac{(x+7)(x-7)}{2(x^2 + x - 42)}$$

GCF + product and sum

$$= \frac{(x+7)(x-7)}{2(x-6)(x+7)} \rightarrow x \neq -7$$

$$= \frac{(x-7)}{2(x-6)} \quad \text{denom} \neq 0$$

$$2(x-6) \neq 0 \rightarrow 2 \neq 0 \quad x-6 \neq 0 \\ \boxed{x \neq 6}$$

$$\therefore \frac{(x-7)}{2(x-6)}, \quad x \neq 6, -7$$

① Simplify the rational expression by factoring since there is addition/subtraction.

② Now, cancel anything THAT IS BEING MULTIPLIED.

③ After finishing simplifying, state your domain restrictions. (You can make each term $\neq 0$ and solve.)

④ Write your final answer (with domain restriction)

NOTE: the domain restriction was only visible AFTER factoring. They are not obvious in expanded form so always factor!!!

Simplify and sketch:

$$y = \frac{(2x+1)(x-1)}{(x-1)}$$

$$y = \frac{(2x+1)(x-1)}{(x-1)}$$

~~(x-1)~~

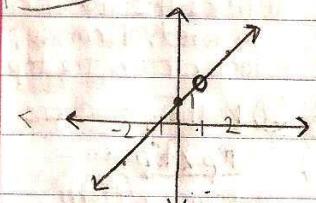
$$y = (2x+1)$$

Domain Restriction

$$x-1 \neq 0$$

$$\boxed{x \neq 1}$$

$$\therefore y = 2x+1 \quad (x \neq 1)$$



① Notice that the question has already been given in factored form. Normally, you would factor first.

② Cancel any common factors BEING MULTIPLIED.

③ Although the simplified LINEAR function has no denominator, you must look at the original expression to identify the domain restriction.

If we had not looked back to our original expression, we wouldn't have known there is a hole in our function after simplifying!!

NOTE: when it says to look at the original to define the domain, look at the fully factored form rather than the expanded form!

(1b) Simplifying with monomial terms

$$\begin{aligned} & \bullet -\frac{70n^2}{28n} = \frac{70}{28} \cancel{n}^2 \quad \text{Cancel } n \\ & -\frac{(2)(5)(7)(n)(m)(n)}{(2)(2)(7)(m)} = \frac{70}{28} \cancel{n}^2 \quad \text{Cancel } n \\ & -(5)(n) = -\frac{5n}{2} \end{aligned}$$

① Expand terms using prime factorization. (Also expand variables that need to be cancelled.)

② Cancel any terms that are being multiplied.

③ Look back at your original expression to restrict the domain.

Even though n isn't in the denominator here, we still need to account for the hole in our original expression.

$$\text{Domain: } (-\infty, 0) \cup (0, \infty)$$

④ To state the domain in interval notation, make sure to use a union symbol.

(1c) Simplifying with Polynomial terms

$$\begin{aligned} & \bullet \frac{v^2 - 5v - 14}{(v^2 + 4v + 4)} = \frac{\text{product}}{\text{sum}} \\ & = \frac{(v+2)(v-7)}{(v+2)(v+2)} \\ & = \frac{(v-7)}{(v+2)} \end{aligned}$$

$$(v+2) \neq 0 \quad \therefore (v-7) \quad , v \neq -2$$

$$[v \neq -2]$$

① Notice here, you have to factor instead of using prime factorization because of addition/subtraction.

② Cancel anything that you are able to. i.e. common binomials.

③ Look at original and restrict domain. Then look at simplified version to make sure you have restricted the domain for it as well.

(2a) Multiplying rational expressions

$$\frac{7n^2(n+4)}{(n-3)(n+4)} \cdot \frac{(n-3)}{(n+8)(n+6)}$$

$$= \frac{7n^2(n+4)(n-3)}{(n-3)(n+4)(n+8)(n+6)}$$

$$= \frac{7n^2}{(n+8)(n+6)}$$

$$\therefore \frac{7n^2}{(n+8)(n+6)}$$

$$\text{if } n \neq 3, -4, -8, -6$$

$$(-\infty, -8) \cup$$

$$(-8, -6) \cup$$

$$(-6, -4) \cup$$

$$(-4, 3) \cup$$

$$(3, \infty)$$

① Note: This question is already factored. Normally, you would factor both the top and bottom.

② Multiply fractions is simply:
top x top
bottom x bottom

③ Cancel common terms/factors

④ Restrict the domain and show how to write in interval notation

NOTE: when finding the **GCF**, pull out **smallest exponent** on term.
 when finding the **LCM**, pull out the **greatest exponent** on term.

(2b) Dividing rational expressions

$$\begin{aligned} & \bullet \frac{6p+27}{18p^2+36p} \div \frac{16p+72}{2p+4} \\ &= \frac{3(2p+9)}{18p(p+2)} \div \frac{8(2p+9)}{2(p+2)} \\ & \quad | p \neq 0 \\ & \quad | p \neq -2 \\ &= \frac{3(2p+9)}{8p(p+2)} \times \frac{2(p+2)}{8(2p+9)} \\ &= \frac{(3)(2p+9)(2)(p+2)}{(18)(p)(p+2)(8)(2p+9)} \quad | 2p+9 \neq 0 \\ & \quad | 2p \neq -9 \\ & \quad | p \neq -4.5 \\ &= \frac{(3)(2)}{(18)(p)(8)} = \frac{6}{144p} \quad (\text{Reduce}) \\ &= \frac{1}{24p} \end{aligned}$$

$$\therefore \frac{1}{24p}, p \neq 0, -2, -4.5$$

① Factor numerators and denominators.

② BEFORE YOU FLIP

TO DIVIDE THE FRACTIONS:

restrict the domain because this is the original

③ FLIP and multiply.

NOTE: if you have multiple fractions to divide, beyond the first fraction, you can flip all and multiply at once.

④ AFTER YOU FLIP:

restrict the domain again.

⑤ Cancel and Simplify

⑥ Look at simplified version to see if any other domain restrictions need to be done.

⑦ Write out final answer

After you flip, see if you can factor before you start cancelling.
(i.e. prime factors.)

(3a) Finding the LCD of monomial terms

$$\bullet \frac{5}{3x^2} + \frac{7}{12x}$$

$$\frac{5}{(3)(x)(x)} + \frac{7}{(3)(4)(x)}$$

$$\text{LCD} = 3(x)(x)(4)$$

$$(2)(x)^2$$

$$\therefore \text{LCD} = 12x^2$$

NOTE: $x \neq 0$

① Write out the denominators using prime factors.

② Look at your first denominator.

Compare it to your second denominator.

Is everything in your second denominator being represented?
If not, put in missing terms/variables.

Finding the LCD of polynomial terms

$$\bullet \frac{5}{a-6} + \frac{7}{a+4}$$

$$\frac{5}{(a-6)} + \frac{7}{(a+4)}$$

$$LCD = (a-6)(a+4)$$

$$\therefore LCD = \underbrace{(a-6)(a+4)}_{\text{DON'T EXPAND}} \\ (\text{unnecessary})$$

① Insert brackets where necessary.

② Both denominators are binomials that are already fully factored. So, find the LCD by comparing your first denominator to your second denominator.

NOTE: $a \neq 6, -4$

$$\bullet \frac{a-2}{a+2} - \frac{a-3}{a^2+4a+4}$$

$$\frac{(a-2)}{(a+2)} - \frac{(a-3)}{(a+2)^2}$$

$$\frac{(a-2)}{(a+2)} - \frac{(a-3)}{(a+2)(a+2)}$$

$$LCD = (a+2)(a+2)$$

$$\therefore LCD = (a+2)^2$$

① Insert brackets where necessary.

② Factor the denominators, if needed.

③ Compare your first denominator to your second denominator to find your LCD. Remember to put in any unrepresented factors!

NOTE: $a \neq -2$

(3b) Example of simplifying a subtraction of rational expressions

$$\bullet \frac{7x}{x^2+x-12} - \frac{2x}{x^2+9x+20}$$

$$\frac{7x}{(x+4)(x-3)} - \frac{2x}{(x+4)(x+5)}$$

$$LCD = (x+4)(x-3)(x+5)$$

$$\frac{7x(x+5)}{(x+4)(x-3)(x+5)} - \frac{2x(x-3)}{(x+4)(x+5)(x-3)}$$

DISTRIBUTE! (NUMERATOR!)

$$\frac{7x^2 + 35x}{LCD} - \frac{2x^2 - 6x}{LCD}$$

SUBTRACTION! Distribute the (-)

$$\frac{7x^2 + 35x - 2x^2 + 6x}{LCD}$$

① Insert brackets as necessary.

② Factor denominators.

③ Compare your first denominator to your second denominator to solve for your LCD.

④ Using your knowledge that $\text{your LCD} = (x+4)(x-3)(x+5)$, write equivalent fractions. Do this by multiplying top and bottom by "missing" factor(s) of the LCD.

⑤ Evaluate the numerators and then CLT

⑥ Restrict domain & end

$$\frac{5x^2 + 41x}{LCD}$$

$$\therefore \frac{5x^2 + 41x}{(x+4)(x-3)(x+5)} ; x \neq -4, 3, -5$$