

## MCR → Radicals

### TOPIC (4) Simplifying Radicals

→ This is breaking up a radical into factors to ultimately end up with the radicand being a prime number.

$$\bullet \sqrt{72}$$

① Think of the largest factors of 72 where one is a perfect square,

$$= \sqrt{36 \times 2}$$

② Take out the root of 36 from under the radical and bring it out in front.

$$= 6\sqrt{2}$$

③ Since 2 is a prime number, the radical is fully simplified.

$$\bullet \sqrt{29}$$

① Think of the largest factors of 29 and look for one being a perfect square.

Because 29 is a prime number, its only factors are 1 and 29. Even though 1 is a perfect square, in this case, it can't be used to simplify the radical.

So  $\sqrt{29}$  is already simplified.

**Radical: ( $\sqrt{\phantom{x}}$ )** The symbol known as a square root symbol.

**Like Radicals:** Radicals that have the same radical sign and the same radicand.

**Radicand:** The number under the radical.

•  $7\sqrt{27}$  ① Factors of 27 where one is a perfect square?  
=  $7\sqrt{9 \cdot 3}$  ② Take the root of 9 out from under the radical and multiply it with the 7 that is already there.  
=  $7 \cdot 3\sqrt{3}$   
=  $21\sqrt{3}$

#### (4b) Adding Radicals

•  $3\sqrt{6} + 2\sqrt{6}$  ① To add radicals, you need to make sure you have like radicals.  
=  $3+2\sqrt{6}$

•  $= 5\sqrt{6}$  ② Then, add the coefficients while keeping the radical and radicand the same.

\* The co-efficients can be positive or negative.

#### Subtracting Radicals

•  $5\sqrt{3} - 11\sqrt{3}$  ① Make sure you have like radicals.

•  $= 5 - 11\sqrt{3}$  ② Subtract coefficients while keeping the radical and radicand the same.

#### Adding/Subtracting Radical Expressions

•  $7\sqrt{8} - 4\sqrt{45} + 7\sqrt{180} + 10\sqrt{18}$  ① In order to add/subtract you need like radicals.  
=  $7\sqrt{4 \cdot 2} - 4\sqrt{9 \cdot 5} + 7\sqrt{36 \cdot 5} + 10\sqrt{9 \cdot 2}$  Since there are none, simplify first.  
=  $7 \cdot 2\sqrt{2} - 4 \cdot 3\sqrt{5} + 7 \cdot 6\sqrt{5} + 10 \cdot 3\sqrt{2}$   
=  $14\sqrt{2} - 12\sqrt{5} + 42\sqrt{5} + 30\sqrt{2}$   
=  $14\sqrt{2} + 30\sqrt{2} - 12\sqrt{5} + 42\sqrt{5}$   
=  $14 + 30\sqrt{2} - 12 + 42\sqrt{5}$   
=  $44\sqrt{2} + 30\sqrt{5}$

•  $6\sqrt[3]{40} + 4\sqrt[3]{135} - 9\sqrt[3]{5}$  ① Simplify the radicand by thinking of a number cubed times one more number to give you the original radicand.

•  $= 6\sqrt[3]{2 \cdot 2 \cdot 5} + 4\sqrt[3]{3 \cdot 3 \cdot 5} - 9\sqrt[3]{5}$  ② Bring out the number that is being cubed and multiply it by the coefficient.  
=  $6 \cdot 2\sqrt[3]{5} + 4 \cdot 3\sqrt[3]{5} - 9\sqrt[3]{5}$   
=  $12\sqrt[3]{5} + 12\sqrt[3]{5} - 9\sqrt[3]{5}$   
=  $12 + 12 - 9\sqrt[3]{5}$   
=  $15\sqrt[3]{5}$

**Conjugate:** An expression where the sign between 2 terms is opposite to another expression.  
ie.  $(3x+1)$  The conjugate of  $(3x+1)$  is  $(3x-1)$ .

(4c) Rationalizing the Denominator: → the process of representing a fraction differently so that the denominator isn't an irrational number.

•  $\frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$  ① Multiply numerator and denominator by the irrational number over the irrational number. \*NOTE  $\frac{\sqrt{2}}{\sqrt{2}} = 1$

$$= \frac{1\sqrt{2}}{2} = \frac{\sqrt{2}}{2}$$

•  $\frac{12}{2-\sqrt{5}}$  ① Recognize that the denominator is a binomial that has an irrational number.

•  $\frac{12}{2-\sqrt{5}} \cdot \frac{2+\sqrt{5}}{2+\sqrt{5}}$  ② Multiply the numerator and denominator by the conjugate of the binomial.

$= \frac{12(2+\sqrt{5})}{(2-\sqrt{5})(2+\sqrt{5})}$  This is a difference of squares in factored form. So, when you evaluate it, you should end up with a difference of squares.

$= \frac{24+12\sqrt{5}}{4-5} = \frac{24+12\sqrt{5}}{-1} = -24-12\sqrt{5}$  ③ Simplify

•  $\frac{1}{\sqrt[3]{x}\sqrt{y}} = \frac{1}{x^{\frac{1}{3}}y^{\frac{1}{2}}}$  ① To make it easier, rewrite  $\sqrt[3]{x}$  as  $x^{\frac{1}{3}}$  and  $\sqrt{y}$  as  $y^{\frac{1}{2}}$ .

$\frac{1}{x^{\frac{1}{3}}y^{\frac{1}{2}}} \cdot \frac{x^{\frac{2}{3}}y^{\frac{1}{2}}}{x^{\frac{2}{3}}y^{\frac{1}{2}}} = \frac{x^{\frac{2}{3}}y^{\frac{1}{2}}}{xy}$  ② Think of what you have to multiply (x) and (y) by to make them to the power of 1.

$= \frac{\sqrt[3]{x^2}\sqrt{y}}{xy}$  ③ Rewrite  $x^{\frac{2}{3}}$  and  $y^{\frac{1}{2}}$  as  $\sqrt[3]{x^2}$  and  $\sqrt{y}$ .

\*NOTE When rewriting  
 $x^{\frac{2}{3}} \cdot "2" = \text{power of number}$        $x^{\frac{2}{3}} = \sqrt[3]{x^2}$   
 $"3" = \text{root of radical}$

#### (4d) Multiplying Radicals

•  $3\sqrt{6x} \cdot 5\sqrt{10x}$  ① Multiply the coefficients with each other and multiply the radicands with each other.  
 also applies to dividing radicals  
 $= 3 \cdot 5 \sqrt{6x \cdot 10x}$   
 $= 15 \sqrt{60x^2}$   
 $= 15 \sqrt{4 \cdot 15 \cdot x^2}$   
 $= 15 \cdot 2 \cdot x \sqrt{15}$   
 $= 30x\sqrt{15}$

② Simplify the radical (don't forget to simplify the variable!)

•  $(4 + \sqrt{5})(11 - \sqrt{5})$  ① Use FOIL the same way you would normally use it.

$$\begin{aligned} &= 44 - 4\sqrt{5} + 11\sqrt{5} - 5 \\ &= 44 - 4\sqrt{5} + 11\sqrt{5} \\ &= 44 - 5 = 39 \end{aligned}$$

② Add/subtract like terms + like radicals

### Dividing Radicals

•  $\frac{\sqrt{2}}{\sqrt{6}} = \frac{\sqrt{1}}{\sqrt{3}}$  ① Simplify if possible  
 $\frac{\sqrt{1} \cdot \sqrt{3}}{\sqrt{3} \cdot \sqrt{3}} = \frac{\sqrt{3}}{3}$  ② Rationalize the denominator by multiplying  $\frac{\sqrt{3}}{\sqrt{3}}$

~~cannot multiply only by  $\sqrt{3}$ .~~

$$\begin{aligned} &\frac{\sqrt{24}}{4\sqrt{3}} \cdot \frac{4\sqrt{3}}{4\sqrt{3}} \quad \text{① Rationalize denominator by multiplying } \frac{4\sqrt{3}}{4\sqrt{3}} \\ &= \frac{4\sqrt{52}}{16 \cdot 3} = \frac{4\sqrt{72}}{48} \quad \text{② Simplify the radical in the numerator} \\ &\frac{4\sqrt{36 \cdot 2}}{48} = \frac{4 \cdot 6\sqrt{2}}{48} = \frac{24\sqrt{2}}{48} \quad \text{③ The division of 48 applies to the coefficient 24. If } \frac{24}{48} \text{ can be reduced (which it can), then reduce.} \\ &= \frac{1\sqrt{2}}{2} = \frac{\sqrt{2}}{2} \end{aligned}$$

### (4)e) Solve root equations with 1 radical

•  $\sqrt{x+12} = x-8$  ① Make sure to have the radical isolated on one side of the equation.

$$(\sqrt{x+12})^2 = (x-8)^2 \quad \text{② To reverse the square root of } (x+12), \text{ square both sides.}$$

$$x+12 = x^2 - 16x + 64 \quad \text{③ Bring terms to one side and simplify by collecting like terms.}$$

$$x^2 - 17x + 52 = 0 \quad \text{④ Factor to get values of } x$$

$$(x-13)(x-4) = 0 \quad \text{⑤ Put both solutions into the original equation to see which one is correct.}$$

$$\sqrt{x+12} = x-8 \quad \sqrt{x+12} = x-8$$

$$\sqrt{13+12} = 13-8 \quad \sqrt{4+12} = 4-8$$

$$\sqrt{25} = 5 \quad \sqrt{16} = -4$$

$$5 = 5 \quad 4 \neq -4$$

X

∴ Solution to equation is  $x=13$

•  $(4 + \sqrt{5})(1 - \sqrt{5})$  ① Use FOIL the same way you would normally use it.

$$\begin{aligned} &= 4 - 4\sqrt{5} + \sqrt{5} - 5 \\ &= 4 - 5 - 4\sqrt{5} + \sqrt{5} \\ &= -1 - 3\sqrt{5} \end{aligned}$$

### Dividing Radicals

•  $\frac{\sqrt{2}}{\sqrt{6}} = \frac{\sqrt{1}}{\sqrt{3}}$  ① Simplify if possible  
 $\frac{\sqrt{1} \cdot \sqrt{3}}{\sqrt{3} \cdot \sqrt{3}} = \frac{\sqrt{3}}{3}$  ② Rationalize the denominator by multiplying  $\frac{\sqrt{3}}{\sqrt{3}}$

•  $\frac{\sqrt{24}}{\sqrt{3}} \cdot \frac{4\sqrt{3}}{4\sqrt{3}}$  ① Rationalize denominator by multiplying  $\frac{4\sqrt{3}}{4\sqrt{3}}$   
 cannot multiply only by  $\sqrt{3}$ .  
 $= \frac{4\sqrt{52}}{16 \cdot 3} = \frac{4\sqrt{72}}{48}$  ② Simplify the radical in the numerator  
 $\frac{4\sqrt{36 \times 2}}{48} = \frac{4 \cdot 6\sqrt{2}}{48} = \frac{24\sqrt{2}}{48}$  ③ The division of 48 applies to the coefficient 24. If 24 can be reduced (which it can), then reduce.  
 $= \frac{1\sqrt{2}}{2} = \frac{\sqrt{2}}{2}$

⑤ ⑥ Rational Exponents: If a base has a fractional exponent, it can be rewritten using a radical.

$$\sqrt[n]{ab} = a^{\frac{b}{n}}$$

•  $a^{\frac{b}{n}}$  ← power of the base  
 $a^{\frac{b}{n}}$  ← index of the radical  $= \sqrt[n]{ab}$

•  $-16^{\frac{3}{4}}$  ① Write expression using a radical

$$-\left(\sqrt[4]{16}\right)^3$$

② Find the  $\sqrt[4]{}$  of 16  
 ③ Apply the exponent "3" to the base and distribute the -1.  
 $- (2)^3 = -8$

**WHEN TO USE ABSOLUTE VALUE SIGNS:** When you are taking the even root of something and you end up with an odd power on your variable, you need to insert absolute value. ODD ROOTS NEVER NEED ABSOLUTE VALUE SIGNS.

$$\text{ie. } \sqrt[4]{x^4} = x' = |x|$$

↑  
odd exponent

•  $(-27)^{\frac{4}{3}}$       ① Write expression using a radical  
 $= (\sqrt[3]{-27})^4 = (-3)^4$       ② Find  $\sqrt[3]{-27}$   
 $= 81$       ③ Apply the exponent "4" to the base

\*Remember to keep the negative sign associated with the -27.

•  $(\frac{4}{9})^{\frac{3}{2}} = \frac{4^{\frac{3}{2}}}{9^{\frac{3}{2}}}$       ① Use the distributive property rule that applies to exponents.  
 $= \frac{(\sqrt{4})^3}{(\sqrt{9})^3} = \frac{(2)^3}{(3)^3} = \frac{8}{27}$       ② Write expression using radicals  
 $\quad \quad \quad$       ③ Find the  $\sqrt{4}$  and  $\sqrt{9}$   
 $\quad \quad \quad$       ④ Apply the exponent "3" to each base.

**Even roots:** When the index of a radical is an even number, we are taking an even root of the radicand.

**Absolute Value:** The value of a number as its distance from zero.

When using rational exponents, there are 3 scenarios that can occur.

① even ✓ positive      In this situation, the answer can be either positive or negative. The absolute value sign must be used.

② odd ✓ positive      In this situation, there can only be one answer so no absolute value signs are needed.

or  
③ odd ✓ negative

③ even ✓ negative      In this situation, there is NO REAL SOLUTION. You can't multiply a negative number an even number of times and still have a negative number.

• Scenario 1:  $\sqrt{49} = 7$   
 but  $(-7)^2 = 49$

$\therefore$  solution is 171.

• Scenario 2:  $\sqrt[3]{-8} = -2$  ( $-2 \cdot -2 \cdot -2 = -8$ )  
 $(2 \cdot 2 \cdot 2 \neq -8)$

$$\sqrt[3]{125} = 5 \quad (5 \cdot 5 \cdot 5 = 125)$$

$(-5 \cdot -5 \cdot -5 \neq 125)$

• Scenario 3:  $\sqrt{-9}$  ( $3 \cdot 3 \neq -9$ )  
 $(-3 \cdot -3 \neq -9)$

• Scenario 1:  $\sqrt{18xy^3} = \sqrt{9 \cdot 2 \cdot x^2 \cdot y^2 \cdot y}$   
 (with variables)  
 $= 3xy\sqrt{2y}$

When simplifying radicals with variables, always take out the absolute value of the variables.

$$= 3|xy|\sqrt{2y}$$

### (5d) Examples of Simplifying using Exponent Laws

$$\frac{(8^{-2}x^4y^{-5})^{-2}}{(4^2x^2y^{-3})^2} = \frac{(8^4x^{-8}y^{10})}{(4^4x^4y^{-6})}$$

① Applying the distributive property exponent law.

$$(4^8x^{-8}y^{10}) = 4^4x^{-12}y^{16}$$

★ ② Because 8 is a multiple of 4, change  $8^4$  to  $4^8$ . This way there are 3 sets of alike bases.

$$(4^4x^{-12}y^{16}) = \frac{4^4y^{16}}{x^{12}}$$

③ Apply the quotient law to all alike bases.

$$= \frac{256y^{16}}{x^{12}}$$

④ Move  $(x^{-12})$  below the fraction line to make it a positive exponent.

$$⑤ Simplify  $4^4$$$

$$\frac{(\sqrt[4]{a^3b^3})^2}{(\sqrt{ab})^3} = \frac{(a^{\frac{3}{4}}b^{\frac{3}{4}})^2}{(a^{\frac{1}{2}}b^{\frac{1}{2}})^3}$$

① Using knowledge of rational exponents, write each term so that it has a fractional exponent.

$$a^{\frac{6}{8}}b^{\frac{6}{8}} = a^{\frac{5}{8}}b^{\frac{5}{8}}$$

② Applying the distributive property law.

$$a^{\frac{1}{8}}b^{\frac{1}{8}}$$

③ Applying the quotient law

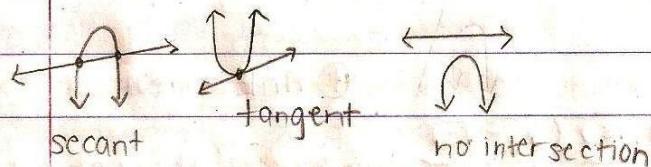
$$\sqrt[8]{a^5}\sqrt[8]{b^5} = \sqrt[8]{a^5b^5}$$

④ Rewrite expression using radicals

⑤ "Combine" both bases because their radicals share the index

### (9a) Ways a line and parabola can meet

- When a line and parabola have 2 points of intersection, the line is **secant** to the parabola.
- When a line and parabola have 1 point of intersection, the line is **tangent** to the parabola.
- When a line and parabola have no points of intersection, there is no intersection between the line and parabola.



### (9b) Using the discriminant to determine if there is an intersection

(Discriminant  $\rightarrow b^2 - 4ac$ )

- If the discriminant equals 0, there is one solution
- If the discriminant equals more than 0, there are 2 solutions
- If the discriminant equals less than 0, there are **No** solutions.

Use the discriminant to determine if/how many points of intersection there are between the linear relation  $y = 0.24x + 7.2$  and the quadratic relation  $y = -0.48x^2 + 4.8x$

$$y = 0.24x + 7.2$$

$$y = -0.48x^2 + 4.8x$$

$$\begin{aligned} 0.24x + 7.2 &= -0.48x^2 + 4.8x \\ -0.48x^2 + 4.8x - 0.24x - 7.2 &= 0 \\ -0.48x^2 + 4.56x - 7.2 &= 0 \end{aligned}$$

$\begin{matrix} a \\ b \\ c \end{matrix}$

$$\begin{aligned} b^2 - 4ac &= (-4.56)^2 - 4(-0.48)(-7.2) \\ &= 20.7936 + 13.824 \\ &= 34.6176 \end{aligned}$$

① Make both equations equal one another

② Collect like terms and make one standard form equation

③ Label terms  $a, b, c$

④ Use discriminant and evaluate

∴ Because the discriminant is more than 0, there are 2 solutions to the linear-quadratic system.

⑨ c) Example of elimination method with circle and ellipse

$$x^2 + y^2 = 7$$

$$4x^2 + 3y^2 = 24$$

$$x^2 + y^2 = 7 \quad (4)$$

$$4x^2 + 4y^2 = 28$$

$$\underline{-4x^2 + 3y^2 = 24}$$

$$\sqrt{y^2} = \sqrt{4}$$

$$y = 2 \quad /y = -2$$

$$x^2 + y^2 = 7$$

$$x^2 + 4 = 7$$

$$x^2 = 7 - 4$$

$$\sqrt{x^2} = \sqrt{3}$$

$$x = \sqrt{3}$$

$$x = -\sqrt{3}$$

① Make a variable in each equation the same.

② Eliminate a variable and solve for the other variable

③ Sub-in this value into an original equation to solve for the remaining variable.

④ Write out P.O.s

$$(2, \sqrt{3}) \quad (2, -\sqrt{3})$$

$$(-2, \sqrt{3}) \quad (-2, -\sqrt{3})$$

⚠ note: the x and y values in this case can be (+) or (-) because when solving we saw that they were being squared.

This is important because realizing this gives us all 4 P.O.s.

(9d) Example of substitution method with two hyperbolas

$$\bullet 4x^2 - y^2 = 5$$

$$xy - 3 = 0$$

$$xy - 3 = 0$$

$$\frac{xy}{x} = \frac{3}{x}$$

$$y = \frac{3}{x}$$

① Isolate a variable in one equation

$$4x^2 - y^2 = 5$$

$$4x^2 - (\frac{3}{x})^2 = 5$$

~~$$(x^2)$$~~ 
$$\frac{4x^2 - 9}{x^2} = 5 \cdot (x^2)$$

$$4x^4 - 9 = 5x^2$$

$$\rightarrow 4x^4 - 5x^2 - 9$$

$$(x^2 + 1)(4x^2 - 9)$$

↓ FACTOR AGAIN

$$(x^2 + 1)(2x - 3)(2x + 3)$$

$$x^2 + 1 = 0 \quad 2x - 3 = 0 \quad 2x + 3 = 0$$

$$x^2 = -1 \quad 2x = 3 \quad 2x = -3$$

X

$$x = 1.5 \quad x = -1.5$$

② Sub-in isolated variable into the other equation

③ Make a standard form equation

④ Factor the standard form equation. (use criss-cross)

⑤ Make each binomial = 0 to find x values

⑥ Sub in each x value to find their corresponding y value for the P1

$$y = \frac{3}{x}$$

$$y = \frac{3}{x}$$

$$y = \frac{3}{1.5}$$

$$y = \frac{3}{-1.5}$$

$$y = 2$$

$$y = -2$$

$$P1(1.5, 2) \quad P1(-1.5, -2)$$