

MHF topic Inequality: "equation" that has  $<$ ,  $>$ ,  $\leq$ ,  $\geq$  instead of an  $=$ .

## Unit 1 - Functions Journal

### (a) Inequalities Solving a linear inequality

goal of solving a linear inequality: to isolate  $x$

Solving a linear inequality is the same as solving a linear equation.

There is only **ONE** rule: if you are multiplying or dividing by a negative number, you have to **FLIP THE SIGN** of the inequality.

$$\begin{array}{l} \bullet x+5 \geq 8 \\ \quad -5 \quad -5 \\ \quad x \geq 3 \end{array} \quad \begin{array}{l} \bullet -x+5 \geq 8 \\ \quad -5 \quad -5 \\ \quad -x \geq 3 \\ \quad \frac{-1}{-1} \quad \frac{-1}{-1} \\ \quad x \leq -3 \end{array}$$

① Subtract 5 from both sides  
② Divide by -1  
FLIP inequality sign!

$$\begin{array}{l} \bullet 2(x+5) \leq 3(5-x) \\ \quad 2x+10 \leq 15-3x \\ \quad -10 \quad -10 \\ \quad +3x \quad +3x \end{array}$$

① FOIL (distribute)  
② Bring all  $x$ 's to one side of the inequality and all integers to the other.

$$\begin{array}{l} 2x+3x \leq 15-10 \\ \frac{5x}{5} \leq \frac{5}{5} \\ x \leq 1 \end{array}$$

③ Collect like terms  
④ Divide both sides by 5

conjunction!

$$\begin{array}{l} \bullet -1 \leq x+3 \leq 8 \\ \quad -3 \quad -3 \quad -3 \\ \quad -2 \leq x \leq 5 \end{array}$$

① Subtract 3 from all sides to isolate  $x$ .  
② Graph the solution set on a number line.  
To graph -2: use a filled in dot because  $x$  is greater than or equal to it.  
To graph 5: use a filled in dot because  $x$  is less than or equal to it.

$$\{x \in \mathbb{R} \mid -2 \leq x \leq 5\}$$

$$[-2, 5]$$

interval notation  
can only be used  
on sets of real  
numbers.

③ Record the solution in set notation.  
Translate this:  $x$  is a member of a set of real numbers such that  $x$  is greater than/equal to -2 and less than/equal to 5.

④ Record the solution set in interval notation.  
(left most point, right most point) Since -2 and 5 can be equal to  $x$ , use square brackets.

disjunction!

Disjunction: where  $x$  can be equal to 2 different sets of values.

Conjunction: where  $x$  has to be equal a specific set of values.

$$3x+5 \leq 10 \quad \text{OR} \quad x+1 > 4$$

$$\begin{array}{r} -5 \\ -5 \end{array} \quad \begin{array}{r} -1 \\ -1 \end{array}$$

$$\frac{3x}{3} \leq \frac{5}{3}$$

$$x > 3$$

$$x \leq \frac{5}{3} \quad \text{OR}$$

$$(1\frac{2}{3})$$



The Union states that  $x$  can be less than/equal to  $\frac{5}{3}$  or greater than 3.

$$\{x \in \mathbb{R} \mid x \leq \frac{5}{3} \text{ OR } x > 3\}$$

is greater than 3.

$$x \leq \frac{5}{3} \quad x > 3$$

$$(-\infty, \frac{5}{3}] \cup (3, \infty)$$

↑  
union symbol

① Solve each inequality (isolate  $x$ )

② Graph each solution set

Because  $x$  can be equal to  $\frac{5}{3}$ , use a filled in dot to graph  $\frac{5}{3}$ . Because  $x$  can't be equal to 3, use a circle outline.

③ Record the solution in set notation.

④ Record the solution in interval notation.

\* because this is a disjunction, record each solution in interval notation then join them with a Union symbol ( $\cup$ ).

\* If you need to use  $-\infty$  or  $\infty$  in interval notation, always use a round bracket because it is impossible to be equal to  $-\infty$  or  $\infty$ .

(1c) Absolute Value: The value of a number as its distance from zero. (The positive value of a number.)

### Solving Absolute Value Equations

THERE ARE 2 SOLUTIONS TO ABSOLUTE VALUE EQUATIONS

$$|2x+3| = 7$$

$$2x+3 = 7 \quad \cancel{(2x+3)} = 7$$

$$\begin{array}{r} -3 \\ -3 \end{array}$$

$$\begin{array}{r} 2x = 4 \\ \frac{2x}{2} = \frac{4}{2} \end{array} \quad \begin{array}{r} 2x+3 = -7 \\ -3 \quad -3 \end{array}$$

$$\boxed{x = 2}$$

$$\frac{2x}{2} = \frac{-10}{2}$$

$$\boxed{x = -5}$$

① Remove absolute value signs to solve for 1st solution.

② Replace absolute value signs with brackets and a coefficient of -1 to solve for 2nd solution.

NOTE \* An absolute value expression cannot equal a negative number. In this case, there are no solutions.

i.e.  $|2x+3| = -7$  NO SOLUTIONS

★ BEFORE SOLVING USING MORE IS OR", "LESS IS NEST",  
MAKE SURE THE ABSOLUTE VALUE Expression IS ISOLATED!  
ON ONE SIDE OF THE INEQUALITY.

### Solving Absolute Value Inequalities

$|x| \geq$  "MORE IS OR" " $|x| \leq$ " "LESS IS NEST"

•  $|3x - 12| \geq 6$

$$\begin{array}{l} 3x - 12 \geq 6 \\ +12 \quad +12 \end{array}$$

$$\begin{array}{l} 3x \geq 18 \\ \hline 3 \quad 3 \end{array}$$

$$\begin{array}{l} x \geq 6 \\ \hline \end{array}$$

$$\begin{array}{l} 3x - 12 \leq -6 \\ +12 \quad +12 \end{array}$$

$$\begin{array}{l} 3x \leq 6 \\ \hline 3 \quad 3 \end{array}$$

$$x \leq 2$$

① Because there is a  $\geq$  sign, we use "more is or."

② For the first solution, remove the absolute value signs and solve for x.

③ For the second solution, remove the absolute value signs and FLIP the inequality. Also, take the opposite of the number. Now solve for x.

∴ the solutions are  $x \geq 6$  OR  $x \leq 2$

•  $|6r - 3| \leq 21$

$$-21 \leq 6r - 3 \leq 21$$

$$+3 \quad +3 \quad +3$$

$$\begin{array}{l} -18 \leq 6r \leq 24 \\ \hline 6 \quad 6 \quad 6 \end{array}$$

$$-3 \leq r \leq 4$$

∴ the solution is

$$-3 \leq r \leq 4.$$

(aka  $r \geq -3$  AND  $r \leq 4$ )

① Because there is a  $\leq$  sign, we use "less is nest."

② Write the negative of the number on the left then write the inequality sign from the given equation.

③ Then remove the absolute value signs around the expression and write it in. After, write another inequality sign. (Same as the one given.)

④ Write the positive of the number on the right.

⑤ Solve the created conjunction.

### (1d) Solving Quadratic Inequalities

•  $x^2 - 7x + 10 > 0$

$$x^2 - 7x + 10 = 0$$

$$(x-2)(x-5) = 0$$

$$x-2=0 \quad x-5=0$$

$$x=2 \quad x=5$$

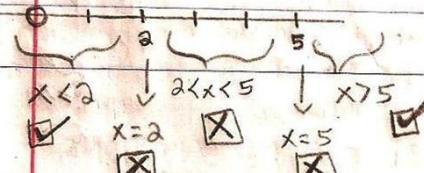
① Turn inequality into an equation  
★ make sure it is equal to 0

② Factor and solve for your x

③ Your x values (roots) are your solutions.

Put your solutions on a number line and then check solutions as well as possible x-values from all intervals.

check:



\* you can check using standard factored form

\* Any single number from an interval that satisfies/doesn't satisfy the inequality means that any number from that interval will have the same result. (i.e. if 0, and number less than 2 satisfied the inequality, all numbers less than 2 will satisfy the inequality.)

check  $x < 2$

$$x^2 - 7x + 10 > 0$$

$$0^2 - 7(0) + 10 > 0$$

$$0 - 0 + 10 > 0$$

$$10 > 0$$

✓

check  $x = 2$

$$x^2 - 7x + 10 > 0$$

$$2^2 - 7(2) + 10 > 0$$

$$4 - 14 + 10 > 0$$

$$-10 > 0$$

✗

check  $2 < x < 5$

$$x^2 - 7x + 10 > 0$$

$$4^2 - 7(4) + 10 > 0$$

$$16 - 28 + 10 > 0$$

$$-12 > 0$$

✗

check  $x = 5$

$$x^2 - 7x + 10 > 0$$

$$5^2 - 7(5) + 10 > 0$$

$$25 - 35 + 10 > 0$$

$$-10 > 0$$

✗

check  $x > 5$

$$x^2 - 7x + 10 > 0$$

$$6^2 - 7(6) + 10 > 0$$

$$36 - 42 + 10 > 0$$

$$-8 > 0$$

✗

✓

**NOTE:** It is mandatory to check all possible solutions because you need to know which inequality sign(s) you will use in your solution.

#### (4e) Solve root equations with 1 radical

$$\sqrt{x+12} = x - 8$$

① Make sure to have the radical isolated on one side of the equation.

$$(\sqrt{x+12})^2 = (x-8)^2$$

$$x+12 = x^2 - 16x + 64$$

② To reverse the square root of  $(x+12)$ , square both sides.

$$x^2 - 16x + 64 - x - 12 = 0$$

$$x^2 - 17x + 52 = 0$$

③ Bring terms to one side and simplify by collecting like terms.

$$(x-13)(x-4) = 0$$

④ Factor to get values of  $x$

$$x = 13 \quad x = 4$$

⑤ Put both solutions into the original equation to see which one is correct.

$$\sqrt{x+12} = x - 8$$

$$\sqrt{x+12} = x - 8$$

$$\sqrt{13+12} = 13 - 8$$

$$\sqrt{4+12} = 4 - 8$$

$$\sqrt{25} = 5$$

$$\sqrt{16} = -4$$

$$5 = 5$$

$$4 \neq -4$$

X

∴ Solution to equation is  $x = 13$

## Solve root equations with 2 radicals

$\sqrt{x+3} = \sqrt{2x+4} - 1$  ① Make sure you have a radical isolated on one side of the equation.

$(\sqrt{x+3})^2 = (\sqrt{2x+4} - 1)^2$  ② Square both sides of the equation to get rid of all radicals.

$(x+3) = (\sqrt{2x+4} - 1)(\sqrt{2x+4} + 1)$  BE CAREFUL WHEN SQUARING BINOMIALS!

$x+3 = 2x+4 - \sqrt{2x+4} - \sqrt{2x+4} + 1$

$x+3 = 2x+4 - 2\sqrt{2x+4} + 1$

$2x+4 - 2\sqrt{2x+4} + 1 = x-3$

$2x - x - 2\sqrt{2x+4} + 4 + 1 = 3$

$x - 2\sqrt{2x+4} + 2 = 3$

③ Add/Subtract like terms AND LIKE RADICALS  
④ Bring all terms to one side and simplify

⑤ Because there is still a radical, isolate it.

$2\sqrt{2x+4} = x+2$  ⑥ Square both sides of the equation to get rid of the radical.

$(2\sqrt{2x+4})^2 = (x+2)^2$

$4(2x+4) = x^2 + 4x + 2$

$8x+16 = x^2 + 4x + 2$

$x^2 + 4x + 2 - 8x - 16 = 0$

$x^2 - 4x - 14 = 0$

$(x-6)(x+2) = 0$

$x=6 \quad x=-2$

⑦ Distribute  
⑧ Bring all terms to one side and simplify

⑨ Put both solutions into the original

equation to see which one is right.

$\sqrt{x+3} = \sqrt{2x+4} - 1$

$\therefore$  the solution to the

$\sqrt{6+3} = \sqrt{2(6)+4} - 1$

equation is  $x=6$ .

$\sqrt{9} = \sqrt{16} - 1$

$\sqrt{1} = \sqrt{0} - 1$

$3 = 4 - 1$

X

✓

COMMON MISTAKES MADE WITH THE SQUARE ROOTS OF NON-MONOMIALS

$(2\sqrt{2x+4})^2$  To get rid of the radical, square the ENTIRE expression.

$= 4(2x+4)$

$\sqrt{(2x+3)^2} = \sqrt{(4x+25)}$

$2x+3 = 2\sqrt{x} \cdot 2\sqrt{x} + 5$

To isolate/solve for x, you would begin by square rooting both sides.

When square rooting the left, it is easy.

When square rooting the right, apply the square root to each term.

(3b)

### Families of equations

What is a family of equations vs. an equation / unique equation?

#### Family of Equations

A family of equations is a set of equations that share characteristics like x-intercepts and degree. The only thing that distinguishes each member of the family is the a-value of each equation. (thinking in terms of factored form)

The zeros of a quadratic function are 5 and 8.

a) Write an equation for the family of quadratic functions with these zeros.

$$f(x) = k(x-5)(x-8), \quad k \in \mathbb{R} \setminus \{0\}$$



b) Write equations for two functions that belong to this family.

$$f(x) = 3(x-5)(x-8) \quad \text{or} \quad f(x) = -\frac{2}{7}(x-5)(x-8)$$

When asked to find a family of equations, your answer should have all roots/ brackets stated with a constant representing the a-value.

### An equation (unique equation)

A unique equation is used to represent a polynomial that must meet a specific condition (i.e. it must pass through a certain point). This is when there is a single a-value that is found by substituting in the point for x and y. This unique equation is one of the many members of that family of equations.

- c) Determine an equation for the member of the family that passes through the point (7, 6).

Sub in (7, 6) for x and y

$$y = k(x-5)(x-8)$$

$$6 = k(7-5)(7-8)$$

$$6 = k(2)(-1)$$

$$\therefore f(x) = -3(x-5)(x-8)$$

$$6 = -2k$$

$$k = -3$$

When asked to find an equation your answer should (already) have all roots/brackets plus a specific a-value that can be found using a point provided.

- State the family of degree 4 polynomials that has been reflected in the x-axis, vertically stretched by 2, with x-intercepts at 4,  $1 \pm \sqrt{3}$  (all multiplicity 1).

info:

$$\text{degree} = 4$$

$$a\text{-value} = -2 \checkmark$$

x-intercepts

$$\begin{array}{l} \checkmark x=4 \\ \checkmark x=1+\sqrt{3} \\ \checkmark x=1-\sqrt{3} \end{array} \quad \left. \begin{array}{l} \text{all are to the power} \\ \text{of one (multiplicity 1)} \end{array} \right\}$$

$$y = -2(x-4)(x-(1+\sqrt{3}))(x-(1-\sqrt{3}))$$

\*Remember to put brackets around binomial roots then distribute the negative!

$$y = -2(x-4)(x-1-\sqrt{3})(x-1+\sqrt{3})(x-t)$$

\*clean-up 2nd and 3rd brackets!

$$y = -2(x-4)\underbrace{(x-1-\sqrt{3})}_{(a-b)}\underbrace{(x-1+\sqrt{3})}_{(a+b)}(x-t)$$

\*factor as difference of squares

$$y = -2(x-4)((x-1)^2 - (\sqrt{3})^2)(x-t)$$

$$y = -2(x-4)(x^2 - 2x + 1 - 3)(x-t)$$

$$(y = -2(x-4)(x^2 - 2x - 2)(x-t),$$

$$t \neq 4$$

this  
is your  
family!

(you don't have to worry about restricting domain for the other bracket because roots are a conjugate and t cannot be a conjugate.)

① Identify the different sources of information given to you and list them to help keep track of what you have/haven't used.

② Write your zeros using brackets/appropriate degrees.

Also, write in your a-value. Make note of what you use.

③ Recognize that at this point, the degree of this polynomial is only 3. To make it degree 4, there needs to be another root. Write in an unknown root and restrict its

that it doesn't equal one of the other brackets to interfere with the multiplicity 1.

- State the family of degree 5 polynomials that pass through points  $(0, 50)$  and  $(-2, -18)$  and have ONLY two real roots with one at  $-1$  (multiplicity 1) and one at  $-5$  (multiplicity 2).

Info:

degree = 5 ✓

points

$(0, 50)$  ✓

$(-2, -18)$  ✓

ONLY TWO ROOTS ✓

$$x = -1 \leftarrow \text{power 1} \quad \checkmark$$

$$x = -5 \leftarrow \text{power 2} \quad \checkmark$$

$$y = (x+1)(x+5)^2$$

\* total degree of 5 is not

satisfied! A quadratic needs to be created! \*

$$y = (x+1)(x+5)^2 (ax^2 + bx + c)$$

NOTE: the question says the polynomial only has 2 roots  $(x+1), (x+5)$ . So, we need to

create a quadratic that has NO roots.

- Before we do this, we need to find values for unknown variables using points provided to us.

Sub-in point  $(0, 50)$

$$y = (x+1)(x+5)^2 (ax^2 + bx + c)$$

$$50 = (0+1)(0+5)^2 (a(0)^2 + b(0) + c)$$

$$50 = (1)(25)(c)$$

$$\frac{50}{25} = \frac{25c}{25}$$

$$\boxed{2=c}$$

- By looking at our list of info, the only thing we haven't used is the fact that this polynomial must only have 2 roots. (Refer back to the NOTE above.) This is when we have to use the discriminant to ensure the quadratic we created has no more roots.

$$(b^2 - 4ac) < 0$$

NOTE: discriminant was used to take this into account.

- List all the information that was given to help map out what you have/haven't used.

- Write your zeros using brackets/appropriate degrees.

\* DONOT write a out in front in this case! Because, you will see that a degree of 2 is missing from the entire polynomial. So, a quadratic needs to be created in the form of  $ax^2 + bx + c$ .

- Sub-in c-value + point  $(-2, -18)$

$$-18 = (-2+1)(-2+5)^2 (a(-2)^2 + b(-2) + 2)$$

$$-18 = (-1)(3)^2 (4a - 2b + 2)$$

$$\frac{-18}{9} = -9(4a - 2b + 2)$$

$$2 = 4a - 2b + 2$$

$$0 = 4a - 2b$$

$$2b = 4a$$

$$\boxed{b=2a}$$

**EXTRA** → steps on solving a polynomial inequality ie.  $\text{polyn} \# 1 > \text{polyn} \# 2$

- ① Make one side equal zero by CLT on one side
- ② Factor this side
- ③ Use roots to test intervals (can have more than 2 intervals to test)  
NOTE: test intervals using original inequality! NOT the one you made "equal to" 0. You use the roots from factored form to test intervals in the original inequality.