

# Unit 0 - Grade 9-10 Review Journal

## Linear Relations

### ① Solve a single equation

•  $3x + 7 = 43$  ① Subtract 7 from both sides.

$$3x + 7 - 7 = 43 - 7$$

$$\frac{3x}{3} = \frac{36}{3}$$
 ② Divide both sides by 3.

$x = 12$  ③ To double-check answer, do a check and substitute solution in for  $x$ .

If the equation is balanced, the solution is correct.

$$8(12) + 7 = 43$$

$$96 + 7 = 43$$

$$103 = 43$$

### Solve Equation with a fraction/fractional coefficients

•  $-\frac{x}{4} - 9 = -5$  ① Multiply both sides by 4.

$$4(-\frac{x}{4} - 9) = 4(-5)$$

$$-x - 36 = -20$$
 ② Add 36 to both sides.

$$-x - 36 + 36 = -20 + 36$$

$$\frac{-x}{1} = \frac{16}{1}$$
 ③ Divide both sides by -1.

$$x = -16$$

•  $\frac{x}{2} - \frac{3x}{4} = \frac{3x}{6} - \frac{5}{6}$  ① Look for a common denominator and then multiply both sides by it.

$$\frac{12}{1} \left( \frac{x}{2} - \frac{3x}{4} \right) = \frac{12}{1} \left( \frac{3x}{6} - \frac{5}{6} \right) \rightarrow \left( \frac{12}{1} \right) \left( \frac{3x}{2} \right) = \frac{36x}{2} = 18x$$

$$12x - 18x = 9x - 10$$
 ② Collect like terms.

$$12x - 18x - 9x = -10$$

$$\frac{-15x}{-15} = \frac{-10}{-15}$$
 ③ Divide by -15 on both sides.

$$x = \frac{2}{3}$$

•  $\frac{2(x-1)}{3} - \frac{1}{5}(2x-3) = 1$  ① Look for a common denominator for the fractions only. Then, multiply both sides by it. It only applies to co-efficients!

$$\frac{15}{1} \left[ \frac{2}{3}(x-1) - \frac{1}{5}(2x-3) \right] = 1 \cdot 15$$

$$10(x-1) - 3(2x-3) = 15$$
 ② Distribute, collect like terms, and then solve for  $x$ .

$$x = 4$$

**Linear System:** A system of linear equations that always has 1 solution, no solution, or an infinite number of solutions.

Solve equation with brackets

•  $3(h-6) = 2(5-2h)$  ① Distribute 3 and 2 into brackets

$$3h - 18 = 10 - 4h \quad \text{② Collect like terms}$$

$$3h + 4h = 10 + 18$$

$$\frac{7h}{7} = \frac{28}{7} \quad \text{③ Divide both sides by 7}$$

$$h = 4$$

(1b) Solve a linear system using elimination

•  $\begin{array}{r} 2x+3y=4 \\ + -2x+7y=16 \end{array}$  ① Choose a variable to eliminate and then add/subtract. (Add equations in this case)

$$0x + 10y = 20$$

$$\frac{10y}{10} = \frac{20}{10} \quad \text{② Divide both sides by 10}$$

$$y = 2 \quad \text{③ Sub-in y value into an original equation and solve for } x.$$

$$2x + 3y = 4$$

$$2x + 3(2) = 4$$

$$2x + 6 = 4$$

$$2x = 4 - 6$$

$$\frac{2x}{2} = \frac{-2}{2}$$

$$x = -1$$

∴ the solution to the linear system

is  $-1, 2$ .

④ Write a ∴ statement

Solve a linear system using substitution

•  $\begin{array}{l} 2x+4y=4 \\ y=x-2 \end{array}$  ① Substitute  $y = x-2$  for  $y$  into first equation and solve for  $x$ .

$$2x + 4(x-2) = 4$$

$$2x + 4x - 8 = 4$$

$$2x + 4x = 4 + 8$$

$$\frac{6x}{6} = \frac{12}{6}$$

$$x = 2 \quad \text{② Sub-in } x \text{ value into either equation to solve for } y.$$

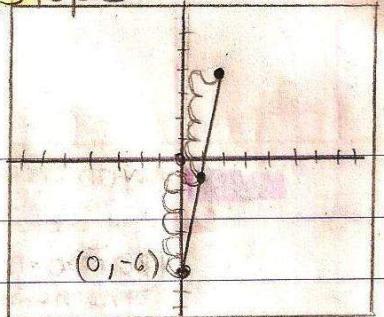
$$y = x - 2$$

$$y = 2 - 2$$

$$y = 0 \quad \text{∴ the solution to the linear system is } 2, 0$$

③ Write a ∴ statement

### Step 3



#### ①c) How to graph lines

$\bullet 5x - y = 6$  ① Put into  $y = mx + b$  form

$$-y = -5x + 6$$

$$y = 5x - 6 \quad \begin{matrix} \uparrow & \uparrow \\ m & b \end{matrix}$$

② Plot your y intercept (your b value) In this case, it is  $0, -6$ .

③ From your y intercept, use your slope (m value) to get more points.

In this case,  $m = 5$  so, go up 5 units, go right 1 unit.

From each point, do this to get another point.

#### ②a) Find equation of a line using slope/y int form

$\bullet (3, -5) (6, 2)$

$$m = \frac{\Delta y}{\Delta x} = \frac{2 - (-5)}{6 - 3} = \frac{7}{3}$$

$$m = \frac{7}{3}$$

$$y = mx + b$$

$$5 = \frac{7}{3}(3) + b$$

$$5 = 7 + b$$

$$5 - 7 = b$$

$$-2 = b$$

$$y = \frac{7}{3}x - 2$$

① Find slope (m value) using the formula

$$m = \frac{y_2 - y_1}{x_2 - x_1} \quad \text{or} \quad \frac{\Delta y}{\Delta x}$$

② Use a point on the line and sub-in for x, y.  
Use the slope and sub-in for m.  
Then, isolate b to find y intercept.

③ Now, plug in the information you have and write your equation.

#### ②b) Find equation of a line using slope/point form

point/slope form:  $y - y_1 = m(x - x_1)$

$\bullet (-3, 5) (2, 8)$

① Sub in a point for x, and y.

$$x_1, y_1$$

$$y - y_1 = m(x - x_1)$$

$$8 - 5 = m(2 - (-3)) \quad \textcircled{a) Calculate slope (m)}$$

$$m = \frac{\Delta y}{\Delta x} = \frac{8 - 5}{2 - (-3)} = \frac{3}{5}$$

③ Now, plug in the information you have and write the equation.

$$y - 8 = \frac{3}{5}(x - 2)$$

④ If you wanted to put it in  $y = mx + b$ , simplify the equation. (Distribute  $\frac{3}{5}$  into brackets then add 8 to both sides.)

### ②(c) How to create an equation from a word problem

- If it costs a factory \$50,000 to make 25 tables and \$180,000 to make 50 tables, what is the cost as a function of the number of tables made? (Write as a linear model)

① Write out the information you know and create a series of points to plot on a graph (for example):

$$\begin{array}{l} 25 \text{ tables} = \$50,000 \\ 50 \text{ tables} = \$180,000 \end{array} \Rightarrow \begin{array}{l} 25, 50\,000 \\ 50, 180\,000 \end{array}$$

### ② Find slope

$$m = \frac{\Delta y}{\Delta x} = \frac{180\,000 - 50\,000}{50 - 25} = \frac{130\,000}{25} \quad m = 1200$$

③ Sub in a point, and slope to solve for b.

$$y = mx + b$$

$$80\,000 = 1200(50) + b$$

$$80\,000 = 60\,000 + b$$

$$80\,000 - 60\,000 = b$$

$$20\,000 = b$$

④ Using the information you found, write out the linear equation.

$$y = 1200x + 20\,000$$

## Algebra

### ③(a) Common Factoring

$$2x^4 - 16x^3$$

→ You use common factoring when you can only factor out a GCF monomial from each term in the polynomial.

① Identify the GCF of both terms. (Look @ exponents also!)

$$\text{GCF} = 2x^3$$

② Factor out (divide) each term by the GCF

$$2x^4 - 16x^3$$

$$2x^3 \quad 2x^3$$

③ Use the distributive property to factor out the GCF and write your factored expression.

$$-2x^3 + 8x^2 - 4x$$

$$\text{GCF} = -2x$$

① Identify a GCF for all terms. In this case, make the GCF a negative.

$$-2x^3 + 8x^2 - 4x$$

② Factor out (divide) each term by GCF.

$$-2x \quad -2x \quad -2x$$

③ Use the distributive property to factor out GCF and write the factored expression

$$-2x(x^2 - 4x + 2)$$

(3b) Factor by Grouping → you factor by grouping when you have more than 2 terms (usually, when you have 4 terms)

$$\bullet 2x^3 - 4x^2 + 3x - 6 \quad \textcircled{1} \text{ Group first 2 terms and last 2 terms}$$

$$(2x^3 - 4x^2)(3x - 6) \quad \textcircled{2} \text{ Look for a GCF in each binomial}$$

$$\text{GCF} = 2x^2 \quad \text{GCF} = 3$$

$$\frac{(2x^3 - 4x^2)}{2x^2} \quad \frac{(3x - 6)}{3 \quad 3} \quad \textcircled{3} \text{ Factor out the GCF from each binomial}$$

$$\frac{2x^2(x - 2)}{x - 2} \quad \frac{3(x - 2)}{x - 2} \quad \textcircled{4} \text{ You will always end up with 2 binomials that are the same. So, now you factor out this binomial common factor.}$$

$$(x - 2)(2x^2 - 3) \quad \textcircled{5} \text{ Write out the factored expression.}$$

$$\bullet 8x^3 + 20x^2 - 2x - 5 \quad \textcircled{1} \text{ Group first 2 terms and last 2 terms}$$

$$(8x^3 + 20x^2)(-2x - 5) \quad \textcircled{2} \text{ Look for a GCF in each binomial}$$

$$\text{GCF} = 4x^2 \quad \text{GCF} = -1 \quad * \text{ If the 1st term is negative, factor out a negative GCF}$$

$$\frac{(8x^3 + 20x^2)}{4x^2} \quad \frac{(-2x - 5)}{-1 \quad -1} \quad * \text{ If there is no GCF, factor out a 1 ALWAYS}$$

$$\frac{4x^2(2x + 5)}{2x + 5} \quad \frac{-1(2x + 5)}{2x + 5} \quad \textcircled{3} \text{ Factor out each GCF from each binomial}$$

$$4x^2(2x + 5) - 1(2x + 5) \quad \textcircled{4} \text{ Factor out binomial common factor}$$

$$(2x + 5)(4x^2 - 1) \quad \textcircled{5} \text{ Write out factored expression}$$

(3c) Difference of Squares → You can identify a difference of squares when you see a perfect square subtracting a perfect square.

$$\bullet 4x^4 - 9 \quad \textcircled{1} \text{ Identify the roots of each term}$$

$$\frac{2x^2}{2x^2} \quad \frac{1}{3 \quad 3} \quad \textcircled{2} \text{ Set up 2 sets of brackets with one having addition and one having subtraction.}$$

$$(2x^2 + ) (2x^2 - ) \quad \textcircled{3} \text{ Insert the roots of the first term into the brackets as the first term.}$$

$$(2x^2 + 3) (2x^2 - 3) \quad \textcircled{4} \text{ Insert the roots of the second term into the brackets}$$

$$\bullet 27x^2y - 48y \quad \textcircled{1} \text{ With any expression, see if you can common factor out a GCF.}$$

$$\text{GCF} = 3y \quad \textcircled{2} \text{ Now, you have a difference of squares that you can factor.}$$

$$3y(9x^2 - 16) \quad \textcircled{3} \text{ Find the roots of } 9x^2 \text{ and } 16, \text{ then set up 2 sets of brackets with addition in one and subtraction in the other.}$$

•  $(y+1)^2 - 36$  ① Think of  $(y+1)$  as one term that is being squared.

$[(y+1)+6][(y+1)-6]$  ② Find the roots of both terms and put roots into brackets with one set being addition and one set being subtraction.

$(y+7)(y-5)$  ③ Now, simplify within each bracket

•  $x^2 - 3$  ① Find the roots of both terms

$x \sqrt{1} \sqrt{3}$  ② Because the root of 3 isn't a whole number, we write it as  $\sqrt{3}$ .

$(x-\sqrt{3})(x+\sqrt{3})$  ③ Place the roots into 2 sets of brackets (one addition and one subtraction)

### ③d) Sum and Product → You use product and sum when the expression/equation has an a-value of 1.

•  $x^2 + 11x + 30$  ① Think of 2 numbers that multiply to (c) and add to (b)

$30 = 5 \times 6$

$11 = 5 + 6$

$(x+5)(x+6)$

② take these 2 numbers and the variable ( $x$ ) and put them into brackets.

Put the factors of  $x^2$  as the first term in each bracket and put the factors that add to 11 and multiply to 30 as the second term in each bracket.

When putting in the numbers, be aware of signs.

•  $x^2 - 4x - 32$  ① Think of 2 numbers that multiply to -32 and add to -4

$-32 = -8 \times 4$

$-4 = -8 + 4$

② Use  $x$  as the first term in each set of brackets and then insert the numbers -8 and 4.

•  $\frac{2y^3}{2y} - \frac{12y^2}{2y} - \frac{80y}{2y}$  ① Common factor out a GCF

$2y(y^2 - 6y - 40)$  ② Now, look for 2 numbers that multiply to -40 and add to -6

$-40 = 4 \times -10$

$-6 = 4 + -10$

③ Use  $y$  as first term in each set of brackets and then insert the numbers 4 and -10.

$(y+4)(y-10)$

### ③ e) Complex Trinomial Factoring → You use this when the a-value in an expression or equation doesn't equal 1.

$$9x^2 + 12x + 4$$

① Recognize that

$$(3x + 2)^2$$

this is a perfect square trinomial.

The 1st and 3rd term are perfect squares and the 2nd term is the product of the roots of the 1st and 3rd term doubled.

② Take the root of the 1st term, the sign from the middle term, and the root of the third term. Put brackets around this binomial and then square it.

$$ax^2 + bx + c$$

•  $3a^2 + 11a + 6$  ① List the factors of 3 forwards and backwards

$$\begin{array}{|c|c|} \hline 1 & 3 \\ \hline 3 & 1 \\ \hline \end{array} \times \begin{array}{|c|c|} \hline 1 & 2 \\ \hline 2 & 1 \\ \hline \end{array}$$

② List the factors of 6 forwards and backwards.  
(It isn't always necessary to write them backwards because it would be repetitive.)

$$6 \times 3 = 18$$

$$1 \times 1 = 1$$

$$18 \times 1 \neq 18 \parallel$$

$$18 + 1 \neq 11$$

$$3 \times 3 = 9$$

$$2 \times 1 = 2$$

$$9 \times 2 = 18$$

$$9 + 2 = 11$$

③ Draw an "x" in the middle

④ When looking at the numbers, their product should be the product of ac and they should add to b.

⑤ Try the first set of numbers. If it doesn't work, try another set.

⑥ Now, put the values into brackets. The first HORIZONTAL line will be the first bracket and the second HORIZONTAL line will be the second bracket.

$$(3 \quad 2)(1 \quad 3)$$

$$(3a \quad 2)(1a \quad 3)$$

$$\begin{array}{|c|} \hline 1 \\ \hline 2a \\ \hline \end{array} \quad \begin{array}{|c|} \hline 1 \\ \hline 9a \\ \hline \end{array}$$

⑦ Since the first term needs to be  $3a^2$ , write in an "a" in each first term (within brackets)

⑧ To determine signs, look at how the 2 numbers have to add to give you the middle term. (11)

$$2a + 9a = 11a$$

So, the signs in both binomials have to be positive.

cross  
cross  
method

so many ways to factor trinomials! (7) methods

1. GCF

~~Note: the following is done the same way as in the previous example but some steps aren't written down. \*~~

① List the factors of 4 and 5

$$\begin{array}{|c|c|} \hline 1 & 2 \\ \hline 2 & 4 \\ \hline 4 & 1 \\ \hline 1 & 5 \\ \hline 5 & 1 \\ \hline \end{array}$$

② Look for 2 sets of numbers whose products are 20 and add to 8.

$$4 \times 5 = 20$$

$$1 \times 1 = 1$$

$$20 \times 1 = 20$$

$$20 + 1 \neq 8$$

$$4 \times 1 = 4$$

$$5 \times 1 = 5$$

$$4 \times 5 = 20$$

$$4 + 5 \neq 8$$

$$1 \times 2 = 2$$

$$5 \times 2 = 10$$

$$10 \times 2 = 20$$

$$10 - 2 = 8$$

③ Put the values of the 2 sets into brackets and determine where to put  $c$  to give  $4c^2$ . Also, determine the signs of the factors to multiply to -20 and add to -8.

$$(2 \quad 1)(2 \quad 5)$$

$$(2c \quad 1)(2c \quad 5)$$

$$-10c$$

It has to be  $2c + (-10c)$   
to add to  $-8c$ .

$$(2c+1)(2c-5)$$

### (5) Exponent Laws: General Rules

**Product Rule:** When bases are alike, you add the exponents

**Quotient Rule:** when bases are alike, you subtract the exponents

**Power Rule:** When something is raised to a power and is being raised to another power, you multiply the exponents.

**Zero Exponent:** When something is raised to the zero exponent, it will always equal 1.

**Negative Exponent:** When something is raised to a negative exponent, make the base a reciprocal. When you make the base a reciprocal, the exponent becomes positive and you can evaluate it.

**Distributive Property Rule:** When distributing an exponent, multiply the exponents of all bases.

Steps for these examples can be found in the rules stated above.

$$(x^6)(x) = x^{6+1} = x^7 \quad \frac{3^6}{3^4} = 3^{6-4} = 3^2 \quad (2^3)^2 = 2^{3 \times 2} = 2^6$$

$$6^0 = 1 \quad \frac{w^4}{w^4} = w^{4-4} = w^0 = 1$$

$$2^{-2} = \frac{1}{2^2} = \frac{1}{4} \quad \frac{2^{-2}}{5^{-2}} = \frac{5^2}{2^2} = \frac{25}{4}$$

$$(x^2 y^5 z^3)^3 = x^{2 \times 3} y^{5 \times 3} z^{1 \times 3} = x^6 y^{15} z^3$$

$$\left(\frac{a^3}{b^2}\right)^4 = \frac{a^{3 \times 4}}{b^{2 \times 4}} = \frac{a^{12}}{b^8}$$

Another way of making negative exponents positive is to move them across the fraction bar.  
(When you have a whole number, taking the reciprocal of the base is doing this.)

Move  $2^{-2}$  below the fraction bar and move  $5^{-2}$  above the fraction bar.

(5b) A common mistake made in the distribution property for non-monomials is that the exponent isn't distributed to all bases.

$$(7x^2 + 5x^3)^2 = 7^{1 \times 2} x^{2 \times 2} + 5^{1 \times 2} x^{3 \times 2} = 7^2 x^4 + 5^2 x^6 = 49x^4 + 25x^6$$

## Quadratic Relations

### ⑥ a) Solving an equation using factoring

$$\bullet \quad x^2 + 5x + 6$$

① Identify how you are going to factor. Because  $a=1$ , use product and sum.

$$(x+2)(x+3)$$

② Find 2 numbers that multiply to 6 and add to 5.

$$x+2=0 \quad x+3=0$$

$$(2, 3)$$

$$x=-2 \quad x=-3$$

③ Take root of first term and put into brackets.  
Then insert 2 and 3.

④ Make each binomial equal 0. Then, isolate x.

⑤ To do a check, all you need to do is substitute  
 $x$  for either -2 or -3. This step isn't always  
necessary but is a good tool/way to see if  
you factored correctly.

★ Equations can be solved by factoring if the equation is factorable.

$$\bullet 3x^2 + 6x = 24$$

$$\frac{3x^2}{3} + \frac{6x}{3} - \frac{24}{3} = 0$$

$$3(x^2 + 2x - 8) = 0$$

$$-8 = 4x^2 \\ 2 = 4 + 2$$

$$3(x+4)(x-2) = 0$$

$$x+4=0 \quad x-2=0$$

$$x=-4 \quad x=2$$

① Make the equation equal 0.

② Common factor out a 3.

③ Do product and sum to factor the equation.

④ Make each binomial equal 0. Then, isolate x.

### Solving an equation using the quadratic formula

\* You use the quadratic formula to solve an equation that can't be factored.

$$\text{Quadratic Formula: } x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\bullet 2x^2 - x - 7 = 0$$

$$a = 2, b = -1, c = -7$$

① Label terms as a, b, c

② Plug values into the equation

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(2)(-7)}}{2(2)}$$

$$x = \frac{1 \pm \sqrt{1 + 56}}{4}$$

$$x = \frac{1 \pm \sqrt{57}}{4}$$

③ Evaluate/Simplify

④ Now, break the equation into "2 parts". Instead of writing the  $\pm$  sign, write out the entire equation with only the (+) sign and with only the (-) sign.

$$x = \frac{1 + \sqrt{57}}{4} \quad x = \frac{1 - \sqrt{57}}{4}$$

⑤ Evaluate (and round answers)

$$x = 2.137 \quad x = -0.887$$

$$x = 2.14 \quad x = -0.89$$

⑥ Sub-in both values of x into the original equation to check. (not always required)

\* In word problems where the quadratic formula is used, 1 of the 2 solutions may need to be "rejected". \*

\* When using the quadratic formula, if the number under the radical (after simplifying) is a 0, then there will only be 1 solution to the equation.

In the same way, if the number under the radical is a negative number, there are no solutions to the equation. \*

↳  $b^2 - 4ac$  is a.k.a the discriminant.

If discriminant = 0  $\rightarrow$  one solution

If discriminant = negative number  $\rightarrow$  no solutions

$a$ -value - greater than 1 = stretched (skinnier)  
less than 1 = compressed (fatter)

### (6b) Completing the Square (Going from standard form to vertex form)

Standard form:  $ax^2 + bx + c$

(a) stretch or compression (c) y intercept

Vertex Form:  $a(x-h)^2 + k$

(b) stretch or compression (h) movement left/right (k) movement up/down

Factored form:  $(x-r)(x-s)$

(r)(s) x intercepts

vertex  
 $r, k$

$$y = x^2 + 8x + 25$$

$$y = (x^2 + 8x) + 25$$

$$y = (x^2 + 8x + (4^2)) - (4^2) + 25$$

$$y = (x+4)^2 - 16 + 25$$

$$y = (x+4)^2 + 9$$

$$\begin{array}{l} x+4=0 \\ x=-4 \end{array}$$

$$\text{vertex: } -4, 9$$

① Group the first 2 terms.

② Common factor out a numerical value if you can.

③ Add half of (b) squared within the brackets and then subtract half of (b) squared outside the brackets.

④ Now there is a perfect square trinomial. So, factor it. Then simplify outside the brackets.

$$y = 4x^2 + 40x + 103$$

$$y = (4x^2 + 40x) + 103 \quad \begin{array}{l} \text{① Group the first 2 terms.} \\ \text{② common factor out (NUMERICAL ONLY)} \end{array}$$

$$y = 4(x^2 + 10x) + 103$$

③ Add half of (b) squared within the brackets and subtract half of (b) squared outside the brackets.

$$y = 4(x^2 + 10x + (5^2)) - (5^2)(4) + 103 \quad \begin{array}{l} \text{④ Because 4 was factored out of the brackets,} \\ \text{b being subtracted needs to be multiplied by 4.} \end{array}$$

$$y = 4(x+5)^2 - 100 + 103$$

$$y = 4(x+5)^2 + 3$$

$$\begin{array}{l} x+5=0 \\ x=-5 \end{array} \quad \begin{array}{l} \text{⑤ Factor the perfect square trinomial and simplify} \\ \text{outside the brackets.} \end{array}$$

$$\text{vertex: } -5, 3$$

### (6c) How to graph Quadratics \*in vertex form

Step pattern: 1, 3, 5, 7, 9, ...

$$y = -3(x+2)^2 + 4$$

$$a = -3 \quad \text{vertex} = -2, 4$$

① Identify the  $a$ -value and plot the vertex as your 1st point.

② Multiply the  $a$ -value by each number in the step pattern. The step pattern works like this:

$$\Rightarrow -3, -9, -15$$

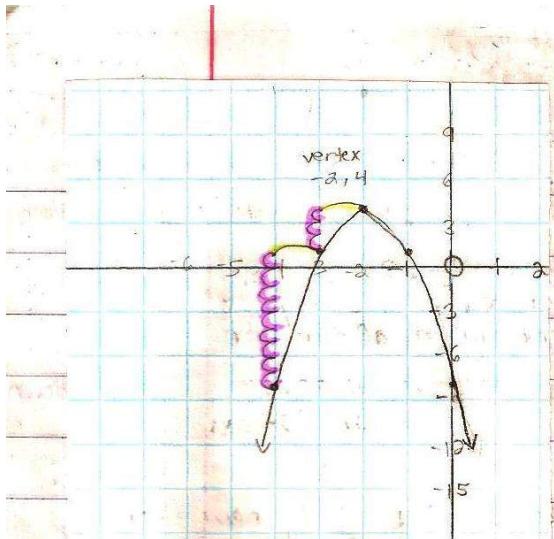
go over 1, up/down 1. go over 1, up/down 3.

Step pattern  
for this  
equation

go over 1, up/down 5.

a negative indicates  
moving down .

1, 3, 5 The step pattern tells you your movement up/down from the vertex.



How to graph  $y = -3(x+2)^2 + 4$

① Start from vertex

② Use step pattern

$$1, 3 \dots (-3) \rightarrow -3, -9 \dots$$

go over 1, down 3

go over 1, down 9

\* Because this is a parabola, do the same movements on both sides

### (6d) Finding a quadratic equation in vertex form

The information needed to find an equation in this form is:

- the vertex

- a point on the parabola (other than vertex)

- Find the equation of a parabola that has its vertex at (1, 4) and has a y-intercept of 1.

$$\text{vertex} = 1, 4$$

① Write out given information.

$$\text{point} = 0, 1$$

② Sub values given into general vertex form equation.

$$y = a(x-h)^2 + k$$

$$1 = a(0-1)^2 + 4$$

③ Solve for your a-value.

$$1 = a(-1)^2 + 4$$

$$1 = a(-1) + 4$$

$$1 = -a + 4$$

$$\begin{aligned} 1 - 4 &= -a \\ -3 &= -a \\ \frac{-3}{-1} &= a \\ 3 &= a \end{aligned}$$

$$\therefore y = 3(x-1)^2 + 4$$

### Finding a quadratic equation in factored form

The information needed to find an equation in this form is:

- the x-intercepts

- a point on the parabola

- Find the equation of a parabola with x-intercepts -3 and 1 and passes through the point (-2, 6).

$$\text{x-intercepts} = -3 \text{ and } 1$$

① Write out the information you have.

$$\text{point} = -2, 6$$

② Sub values into general factored form equation

$$y = a(x-r)(x-s)$$

③ Solve for your a-value.

$$6 = a(-2+3)(-2-1)$$

$$6 = a(1)(-3)$$

$$6 = a(-3)$$

$$\begin{aligned} 6 &= -3a \\ \frac{6}{-3} &= a \\ -2 &= a \end{aligned}$$

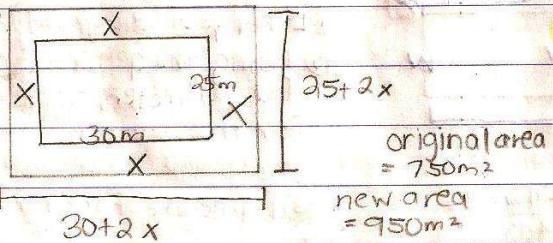
$$\therefore y = -2(x+3)(x-1)$$

**Note:** If you wanted to write an equation in standard form, just go to vertex form/factored form, then distribute/simplify. (i.e. use FOIL)

**NOTE:** your  $a$ -value will be the same in all forms of quadratic equations.

① Solving a word problem where you have to solve for  $x$

- A rectangular building measuring 30m by 25m is going to have its area increased by  $200\text{m}^2$  by adding a strip of uniform width to all four sides. Determine the width of the strip.



① Draw a diagram to show the information given

$$\begin{aligned} \text{original area} \\ = 750\text{m}^2 \end{aligned}$$

$$\begin{aligned} \text{new area} \\ = 950\text{m}^2 \end{aligned}$$

② Using the new area information, write an equation to represent it.

$$\text{new area} \rightarrow A = lw \quad 950 = (30+2x)(25+2x)$$

$$950 = (30+2x)(25+2x)$$

$$950 = 750 + 60x + 50x + 4x^2$$

$$0 = 750 + 60x + 50x + 4x^2 - 950$$

$$\text{new area} \rightarrow 0 = 4x^2 + 110x - 200$$

③ FOIL this, simplify, and make it equal 0. (Make it into a standard form equation.)

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-110 \pm \sqrt{(110)^2 - 4(4)(-200)}}{2(4)}$$

$$x = \frac{-110 \pm \sqrt{12100 + 3200}}{8}$$

$$x = \frac{-110 \pm \sqrt{15300}}{8}$$

$$x = \frac{-110 + \sqrt{15300}}{8} \quad x = \frac{-110 - \sqrt{15300}}{8}$$

$$x = 1.711$$

$$x = -29.211$$

$$x = 1.71$$

$$x = -29.21$$

∴ the width of the strip  
is 1.71m.

④ Find the value of  $x$  by factoring/quadratic formula.

⑤ Because a width can't be a -29.21m, we reject this  $x$ -value and accept 1.7m.

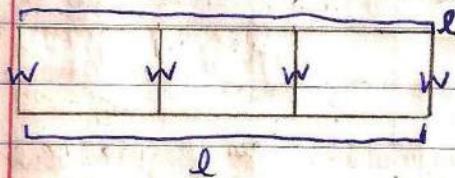
⑥ Write therefore statement  
→ you can check your answer by substituting  $x$  for 1.71 in the equations

$$950 = (30+2x)(25+2x)$$

$$0 = 4x^2 + 110x - 200$$

(7b) Solving a word problem where you have to solve for the MAX value

- A pigpen is to be enclosed as shown. If the total fencing available is 800m, what dimensions will produce a maximum area?



You would sub in the 2-value into the area equation because you are being asked to solve for dimensions that give you a max area. (Unknown max area.)

$$P = l + w \quad 800 - 4w = \frac{2l}{2}$$

$$800 = 2l + 4w \quad 400 - 2w = l$$

$$\begin{aligned} A &= lw \\ A &= (400 - 2w)w \\ A &= 400w - 2w^2 \\ A &= -2w^2 + 400w \\ A &= -2(w^2 - 200w) \end{aligned}$$

$$A = -2(w^2 - 200w + (-100^2)) - (-100^2)(-2)$$

$$A = -2(w - 100)^2 + 20000$$

$$\text{vertex} = 100, 20000$$

$$\begin{aligned} 400 - 2w &= l \\ 400 - 2(100) &= l \end{aligned}$$

$$\begin{aligned} 400 - 200 &= l \\ 200 &= l \end{aligned}$$

∴ dimensions of the pig pen are

$$l = 200 \text{ m} \quad w = 100 \text{ m}$$

These dimensions give a max area of  $20000 \text{ m}^2$

① Label your diagram

② Using the general formula for perimeter, create a formula for this scenario.

③ Isolate one of the variables.

④ Use substitution to solve for the max area:

So, sub in  $400 - 2w$  for  $l$  into the equation for area.

$$A = lw$$

⑤ Begin completing the square to find your vertex.

⑥ At the maximum point,  $w = 100$  and the max area ( $y$ ) = 20000

⑦ To find  $l$ , sub in  $w = 100$  into  $400 - 2w = l$ .

⑧ Write ∴ statement

(8a) Setting up a revenue word problem

- If a gym charges its members \$300 per year to join, they get 1000 members. For each \$2 increase in price they can expect to lose 5 members. How much should the gym charge to maximize its revenue? What is the gym's maximum revenue?

$$\text{Revenue} = (\# \text{ of members})(\text{price})$$

⑨ Recognize what the term "revenue" means in relation to the problem.

general revenue equation

$$\rightarrow \text{Revenue} = (\text{quantity sold})(\text{price})$$

(# of members) (price)

1000 members  $\rightarrow \$300$

-5 members  $\rightarrow + \$2$

let  $R$  = revenue

let  $x$  = # of price increases

$$R = (1000 - 5x)(300 + 2x)$$

↓  
FOIL  
↓

complete the  
square

↓  
vertex

② Write down the information you know from the problem

③ Translate what you know into an equation. This step involves assigning variables before you begin.

④ Because the question asks you for the maximum revenue, you need to find the vertex.

TO DO THIS:

- FOIL  $R = (1000 - 5x)(300 + 2x)$
- COMPLETE THE SQUARE with the standard form equation you get from FOILING

⑤ Your vertex  $(x, R)$  tells you 2 things.

(x) tells you number of price increases for your max revenue

(R) tells you your max revenue

⑥ To find the price the gym charges @ max revenue, sub in

$(x)$  into  $(1000 - 5x)$

### (8)b) Setting up a profit word problem

A skateboard manufacturer lists their revenue and cost as  $R = -x^2 + 24x$  and  $C = 12x + 28$ . Determine an equation for profit and find the value of  $x$  that maximizes profit.

General profit equation

$$\Rightarrow \text{Profit} = \text{Revenue} - \text{Costs}$$

$$\text{Revenue} = -x^2 + 24x$$

$$\text{Costs} = 12x + 28$$

let  $P$  = profit

let  $x$  = selling price

$$P = (-x^2 + 24x) - (12x + 28)$$

$$P = -x^2 + 24x - 12x - 28$$

$$P = -x^2 + 12x - 28$$

This equation represents profit

↓  
Complete the square

$$y = a(x-h)^2 + k$$

$$x - h = 0$$

$$x = h$$

① Recognize what the term "profit" means in relation to the problem.

② Write down the information you know.

③ Translate what you know into an equation. Also, assign variables before you begin.

④ Simplify this equation and collect like terms.

⑤ To find the value of  $x$  that maximizes profit, complete the square with the standard form equation

$$P = -x^2 + 12x - 28$$

Then, make  $(x-h) = 0$  and isolate  $x$ . This gives you the value of  $x$  that maximizes profit.

(8)c) Set up and solve a projectile motion question

- A person standing on top of a bridge 8m high throws a ball upward at an initial velocity of 43m/sec. Assuming the ball is thrown 2m above where the person is standing, find the time it will take for the ball to fall into the river.

$$h = -\frac{1}{2}gt^2 + v_0 t + k$$

$h$  = height

$g$  = gravity = 9.8 m/s<sup>2</sup>

$v_0$  = initial velocity

$k$  = initial height

$t$  = time in seconds

→ To find when the ball falls into the river, I need the x-intercepts. So, I need to put the equation in factored form.  
I know, at this point  $h=0$

$$0 = -\frac{1}{2}(9.8)(t^2) + 43(t) + 8$$

$$0 = -4.9t^2 + 43t + 8$$

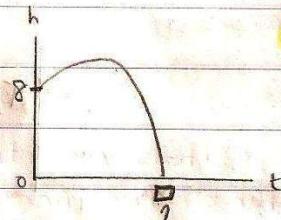
(use quadratic formula)

$$t = 8.9 \quad t = -0.18$$

✓

x

↑  
can't have  
a negative  
time



① Identify formula that you will be using and understand what each variable means.

② Draw a diagram of information.

③ Identify what you need to do.

④ plug all information into the equation. Then simplify.

⑤ Use the quadratic formula to factor the standard form equation you have.

⑥ Write therefore statement

∴ It takes the ball 8.9 seconds to fall into the river.

(8)d) Setting up a distance, time, rate question

general equations

$$\Rightarrow \text{distance} = (\text{rate})(\text{time})$$

$$\text{time} = \frac{\text{distance}}{\text{rate}}$$

$$\text{rate} = \frac{\text{distance}}{\text{time}}$$

- In a dog-sled race, the distance was 225 km. To the checkpoint, weather conditions were excellent. Coming back the other way, weather conditions decreased speed by 10 km/h. The total race took 8.5 hours. What was the speed of racers on the way to the checkpoint?

looking for: rate =  $x$

To checkpoint

$$\text{Distance} = 112.5 \text{ km} \quad \text{rate} = x$$
$$\text{Time} = \text{distance} \div \text{rate} \quad \text{Time} = \frac{112.5}{x}$$

① Create charts to present information that you know. Also, identify what you are solving for. (And assign a variable)

From checkpoint

$$\text{Distance} = 112.5 \text{ km} \quad \text{rate} = x - 10$$

$$\text{Time} = \text{distance} \div \text{rate} \quad \text{Time} = \frac{112.5}{x - 10}$$

Total

$$\text{Distance} = 225 \text{ km} \quad \text{rate} = x$$

$$\text{Time} = 8.5 \text{ hours (8 hr 30 min)}$$

$$\frac{112.5}{x} + \frac{112.5}{x - 10} = 8.5$$

② Create an equation

total time of race

↑  
Time to  
checkpoint

↑  
Time from  
checkpoint

③ Solve for  $x$

= find a common denominator

- FOIL

- factor the standard form equation using quadratic formula

# Trigonometry

## ⑩ a) Angle Relationships

a) Acute angles → Greater than  $0^\circ$  but less than  $90^\circ$

b) Right angles → Equal to  $90^\circ$

c) Obtuse angles → Greater than  $90^\circ$  but less than  $180^\circ$

d) Reflex angles → Greater than  $180^\circ$  but less than  $360^\circ$

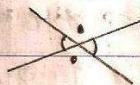
Angle Relationships that come from parallel lines and a transversal

$\rightarrow \begin{matrix} 1/2 \\ 3/4 \end{matrix} \rightarrow$  (1, 4) (2, 3) (5, 8) (6, 7) opposite angles are equal  
(3, 7) (4, 8) (1, 5) (2, 6) corresponding angles are equal

$\rightarrow \begin{matrix} 5/6 \\ 7/8 \end{matrix} \rightarrow$  (3, 6) (4, 5) alternate interior angles are equal

(4, 6) (3, 5) co-interior angles equal  $180^\circ$

## Patterns to recognize angle properties



opposite angles



corresponding angles

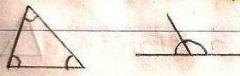


alternate interior angles

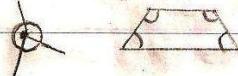


co-interior angles

## More Angle Properties



angles in a triangle or on a line add up to  $180^\circ$



angles around a point or in a quadrilateral add up to  $360^\circ$

### (10b) Similar Triangles

→ triangles that are the same shape but not the same size.  
(angles are all the same but side lengths are different)

- \* In order for them to be similar, corresponding sides must ALL be proportional.
- The two triangles are similar. Determine the measure of DF and EF.

K-value  
big → small  
divide!  
small → big  
multiply!

$$\begin{array}{ccc} \text{A} & \triangle B & \triangle C \\ AC = 15 & BC = 9 & DE = 6 \\ AB = 10 & & \end{array}$$

$$\begin{array}{l} AC = DF \\ [AB = DE] \rightarrow \text{we know } AB = 10 \\ BC = EF \end{array}$$

$$\frac{AB}{DE} = \frac{10}{6} = \frac{5}{3} \quad \text{K-value} = \frac{5}{3}$$

$$\begin{array}{l} AC = DF \\ 15 \div \frac{5}{3} = DF \end{array}$$

$$\begin{array}{l} BC = EF \\ 9 \div \frac{5}{3} = EF \end{array}$$

$$9 = DF \quad 5 \cdot 4 = EF$$

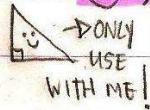
① List all corresponding sides and

② Look for a set that you know the values of both sides.

③ Find the proportion of this side (K-value) by dividing  $\frac{AB}{DE}$  ( $\frac{10}{6}$ )

④ Take this K-value and use it to find the side length of corresponding sides in each pair.

### (10c) SOH CAH TOA



→ only use with me!

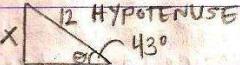
SOH →  $\sin \theta = \frac{\text{opposite}}{\text{hypotenuse}}$    CAH →  $\cos \theta = \frac{\text{adjacent}}{\text{hypotenuse}}$    TOA →  $\tan \theta = \frac{\text{opposite}}{\text{adjacent}}$

SOH CAH TOA can be used to find sides/angles in RIGHT ANGLE triangles only.

$\theta$  represents the angle given/the angle you are finding.

SOH, CAH, TOA are also known as trigonometric ratios

### Using SOH to find a side

OPPOSITE 

$$\text{SOH} = \sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\sin 43^\circ = \frac{x}{12}$$

$$12(\sin 43^\circ) = x$$

$$12(0.68) = x$$

$$8.16 = x$$

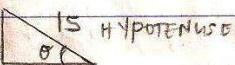
① Label your triangle to determine what trig ratio you are going to use. (Use SOH because you know the opposite and hypotenuse)

② Plug in information

③ Cross multiply to get rid of division of 12

④ Solve for x

### Using CAH to find an angle

 15 HYPOTENUSE  
8 Adjacent

$$\text{CAH} = \cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\cos\theta = \frac{8}{15}$$

$$\cos^{-1}(\cos\theta) = \cos^{-1}\left(\frac{8}{15}\right)$$

$$\theta = 60.3$$

$$\theta = 60^\circ$$

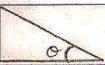
① Label your triangle to determine what trig ratio you are going to use.

(Use CAH because you know the adjacent and hypotenuse)

② Plug in information

③ Apply the inverse cos ( $\cos^{-1}$ ) to both sides to isolate  $\theta$

### Using TOA to find an angle

opposite 7  7  
adjacent 4

$$\text{TOA} = \tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan\theta = \frac{7}{4}$$

$$\tan^{-1}(\tan\theta) = \tan^{-1}\left(\frac{7}{4}\right)$$

$$\theta = 60.2$$

$$\theta = 60^\circ$$

① Label your triangle to determine which trig ratio you have to use (Use TOA because you know the opposite and adjacent.)

② Plug in information

③ Apply the inverse tan ( $\tan^{-1}$ ) to both sides to isolate  $\theta$

★ WHEN SOLVING FOR AN ANGLE, ALWAYS APPLY THE INVERSE TO ISOLATE  $\theta$ . NEVER DIVIDE. ★

### (D) Sine and Cosine Laws

$$\text{Sine law: } \frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

use the sine law when you are given any of the following:

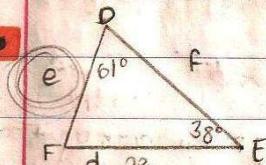
- 2 angles and a side
- 2 sides and an angle

$$\text{Cosine law: } a^2 = b^2 + c^2 - 2bc(\cos A)$$

use the cosine law when you are given any of the following:

- an angle trapped by 2 sides
- 3 sides and no angles

Solving  
for a  
Side using  
Sine law



In this triangle, find the measure of side e.

$$\frac{\sin D}{d} = \frac{\sin E}{e}$$

$$\frac{\sin 61}{23} = \frac{\sin 38}{e}$$

$$e(\sin 61) = 23(\sin 38)$$

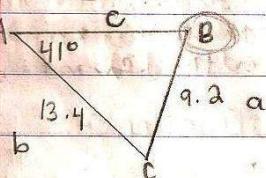
$$\frac{e}{\sin 61} = \frac{23}{\sin 38}$$

$$e = \frac{23(\sin 38)}{\sin 61}$$

$$e = 16.1$$

$$e = 16$$

Solving  
for an  
angle  
using sine  
law



In this triangle, find the angle of B.

$$\frac{\sin A}{a} = \frac{\sin B}{b}$$

$$\frac{\sin 41}{9.2} = \frac{\sin B}{13.4}$$

$$\frac{13.4(\sin 41)}{9.2} = \frac{9.2(\sin B)}{9.2}$$

① Label sides of the triangle. (Sides are written in lowercase letters and are opposite of the angle.)

② Determine which law you need to use.

Then, plug in the information you know.  
(you know 2 angles and 1 side. So use sine law).

③ cross multiply to get rid of the division

④ Isolate e (divide sin 61)

⑤ Solve for e

① Label sides of your triangle

② Determine what law to use (You know 2 sides and an angle, so use sine law)  
Now, plug in the information you know

③ Cross multiply to get rid of the division

④ Divide by 9.2 to isolate (sin B)

$$\frac{B \cdot 4(\sin 41)}{9.2} = \sin B$$

⑤ Evaluate and solve for  $\sin B$

$$0.9555... = \sin B$$

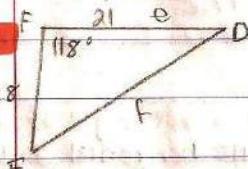
⑥ To isolate  $B$ , apply the inverse  $\sin^{-1}$ . This gives you angle  $B$ .

$$(\sin^{-1}) 0.9555... = \sin B (\sin^{-1})$$

$$72.8 = B$$

$$73^\circ = B$$

Solving  
for a  
side  
using  
cosine  
law



In this triangle,  
find side  $F$ .

$$f^2 = d^2 + e^2 - 2de(\cos F)$$

$$f^2 = 18^2 + 21^2 - 2(18)(21)(\cos 118)$$

$$f^2 = 324 + 441 - 756(\cos 118)$$

$$f^2 = 765 - 756(-0.4695)$$

$$f^2 = 765 + 354.942$$

$$f^2 = 1119.942$$

$$f = 33.4$$

$$f = 33$$

① Label the sides of your triangle

② Determine what law you have to use. (You have an angle trapped by 2 sides. Use the cosine law)

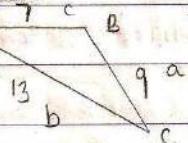
Now, plug in the information you know.

③ Evaluate and solve for  $f$

\*Note -ade is the coefficient of  $(\cos F)$

Solving

for an  
angle  
using  
cosine  
law



In this triangle, find angle  $C$ .

① Label the sides of your triangle

② Determine what law you have to use (you have 3 sides and no angles so you have to use the cosine law)

Then, plug in the information you know

$$c^2 = a^2 + b^2 - 2ab(\cos C)$$

$$7^2 = 9^2 + 13^2 - 2(9)(13)(\cos C)$$

$$49 = 81 + 169 - 234(\cos C)$$

$$49 - 81 - 169 = -234(\cos C)$$

$$\frac{-201}{-234} = \frac{-234(\cos C)}{-234}$$

$$(\cos^{-1}) 0.85... = \cos C (\cos^{-1})$$

③ Evaluate

④ Isolate  $-234(\cos C)$

⑤ Divide by  $-234$  to solve for  $\cos C$

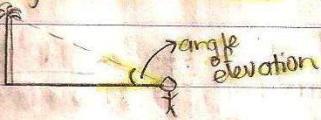
⑥ Apply the inverse  $\cos(\cos^{-1})$  to get angle  $C$ .

$$30.7 = C$$

$$31^\circ = C$$

\* angle of elevation = angle of depression

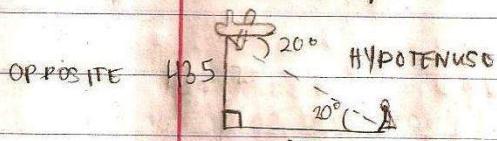
(10e) Angle of Elevation → the angle formed between the line of sight and a horizontal line. The angle formed is above the horizontal line.



Angle of Depression → the angle formed between the line of sight and a horizontal line. The angle formed is below the horizontal line.



• An airplane over an ocean spots a buoy at a  $30^\circ$  angle of depression. If the plane is 435m above water, what is its horizontal distance from the buoy in metres?



$$\text{TOA } \tan \theta = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan 20 = \frac{435}{x}$$

$$x(\tan 20) = 435$$

$$\frac{x}{\tan 20} = \frac{435}{\tan 20}$$

$$x = \frac{435}{\tan 20}$$

$$x = 1195.1$$

$$(x = 1195)$$

① Draw a picture to present the information that you know.

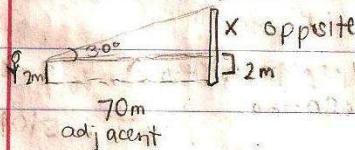
② Determine what trig ratio/law you have to use.  
(you know the opposite and you are looking for the adjacent. Use TOA)

③ Cross multiply then solve for x

④ Write ∴ statement

∴ the horizontal distance of the plane from the buoy is 1195m.

• To find the height of a pole, a surveyor moves 70m away from the base of the pole and then with a transit 2m tall, measures the angle of elevation to the top of the pole to be  $30^\circ$ . How tall is the pole?



$$\text{TOA } \tan \theta = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan 30 = \frac{x}{70}$$

$$70(\tan 30) = x \quad 40.4 = x$$

① Draw a picture to show what you already know

② Determine what trig ratio/law you have to use. (since you know adjacent and are looking for opposite, use TOA)

③ Cross multiply then solve for x

$$x = 40 + 2 \quad x = 42$$

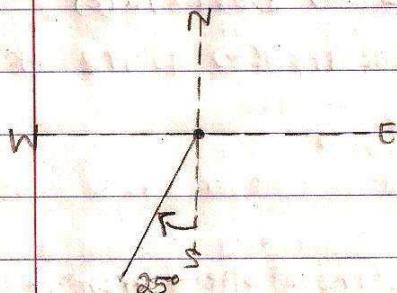
∴ the pole is 42m tall

④ Since the transit was 2m, add 2m  
to  $x = 40$

⑤ Write ∴ statement

### How to draw bearings

- Draw S25°W



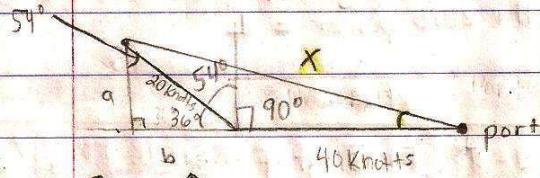
① Draw compass rose + label

② The "S" tells us to draw our line going down (south)

③ 25°W tells us to measure 25° from the "S" line towards the left (towards west)

### Word Problem with bearings

- A ship leaves port at noon headed due west at 20 knots per hour. At 2pm, it changes course to N54°W. Find the ship's bearing and distance at 3pm.



① Draw a picture to see the information you have been given.

② Identify what you need to find.

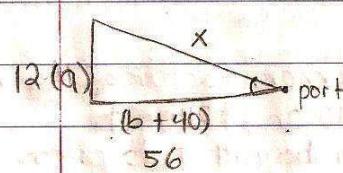
③ Begin solving

④ After find  $a+b$ , create a new large right angle triangle

⑤ use  $a^2 + b^2 = c^2$  to find the distance of the ship at 3pm

use TOA to find the angle bearing.

⑥ Write ∴ statement



$$\text{TOA: } \tan \theta = \frac{12}{56} (\tan^{-1})$$

$$\theta = \frac{12}{56}$$

$$\theta = 12^\circ$$

∴ The ship's distance

at 3pm was 56 knots and its bearing was N12°W.

$$12^2 + 56^2 = c^2$$

$$144 + 3136 = c^2$$

$$\sqrt{3190} = c$$

$$56.4 = c$$

$$56 = c$$

## Common Mistakes

### (II) a) Powers of negative numbers

•  $(-3)^2 = ?$       ① Square  $(-3) \rightarrow$  Do:  $(-3) \times (-3)$

$(-3) \times (-3) = +9$       (we do this because brackets are around  $-3$ )  
 $\rightarrow (-3)^2 = 9$

•  $-3^2 = ?$       ① Square  $3 \rightarrow$  Do:  $(3) \times (3)$

$(-) (3 \times 3)$       ② Distribute the  $(-)$  to the positive  $9$ .  
 $(-) (9) = -9$   
 $\rightarrow -3^2 = -9$

### (II) b) zero Powers

•  $6^0 = \frac{6^2}{6^2} = \frac{6 \times 6}{6 \times 6}$       ① Apply the quotient exponent law  
to this situation

$$= 6^{2-2} = 1 \times 1$$

$$= 6^0 = 1$$

Therefore, anything that is raised to the zero power = 1

### (II) c) Dividing by zero

It's possible to divide zero. ~~It isn't possible to divide another number by zero.~~  
 $0 \div 5 = 0$        $\frac{0}{6} = 0$

$5 \div 0 = \text{UNDEFINED}$

$\frac{6}{0} = \text{UNDEFINED}$

• Look at the following manipulation of an equation:

$$\begin{aligned} a &= b \\ a^2 &= ab \quad | \text{x both sides by } a \\ a^2 - b^2 &= ab - b^2 \quad | - b^2 \text{ from both sides} \\ (a+b)(a-b) &= b(a-b) \quad | \text{factor} \\ (a-b) &\quad (a-b) \quad | \text{divide out a binomial common factor of } (a-b) \\ &\quad \text{STOP! you cannot do anything to this equation beyond this point.} \end{aligned}$$



We were told  $a=b$   
 $so: (a-b) = 0$

Therefore, when we divided a binomial common factor of  $(a-b)$  we were dividing by zero. This produces an undefined answer.

### (11d) Fraction Mistakes

Adding/Subtracting: you need to find the LCD of the 2 fractions. To do this, multiply or divide fractions so that their denominator is the same. Then, add/subtract the numerators to get your sum/difference. (At the end, if you can, reduce your fraction.)

Multiplying: you multiply the numerators, you multiply the denominators to get your product. (At the end, reduce your fraction if you can.)

Dividing: Take the reciprocal of the second fraction. Then, multiply the numerators and denominators to get your quotient. (At the end, reduce your fraction if you can.)

$$\bullet \frac{2}{3} + \frac{3}{5} \quad \text{LCD} = 15 \quad \textcircled{1} \text{ Find LCB}$$

$$\frac{2}{3} \times 5 = \frac{10}{15} \quad \frac{3}{5} \times 3 = \frac{9}{15} \quad \textcircled{2} \text{ Make equivalent fractions}$$

$$\frac{10}{15} + \frac{9}{15} = \frac{19}{15} \quad \textcircled{3} \text{ Add numerators}$$

$$\bullet \frac{2}{3} \times \frac{3}{5} = \frac{6}{15} \quad \textcircled{1} \text{ Multiply numerators and denominators}$$

$$\bullet \frac{2}{3} \div \frac{3}{5} \quad \textcircled{1} \text{ Take the reciprocal of the 2nd fraction}$$

$$\frac{2}{3} \times \frac{5}{3} = \frac{10}{9} \quad \textcircled{2} \text{ Multiply the numerators and the denominators.}$$

### (11e) Exponent Mistakes

$$\bullet x^4 \times x^5 = x^{4+5} \quad \textcircled{1} \text{ Use the product rule since the 2 terms are being multiplied}$$

$$\bullet (x^2)^3 = x^{2 \times 3} \quad \textcircled{1} \text{ Use the power of a power rule since the term raised to the power of 2 is being raised to another power}$$

When deciding how to deal with exponents, first understand what operations are being carried out. Be able to identify things like multiplication, division, exponentiation, etc.

### (11) f) Making Up Rules

•  $\frac{a+b}{c}$  is equal to  $\frac{a}{c} + \frac{b}{c}$        $\frac{a}{b+c}$  is NOT equal to  $\frac{a}{b} + \frac{a}{c}$

\* if you are unsure of when/when not to split up fractions, sub in values for each variable and see if the expressions are equal

→ the fraction  $\frac{a}{b+c}$  is saying: a divided by the sum of b and c = ?

∴ you can't break this fraction up.

→ the fraction  $\frac{a+b}{c}$  is saying: the sum of a and b divided by c = ?

∴ you can break this fraction up because you would still be dividing a and b by c and THEN adding them.

You are essentially performing the same operations in a different order.

### (11) g) Cancelling Incorrectly

"cancel out" → cancel out common factors of the top and bottom lines

To cancel out in a fraction, make sure all terms above the fraction line have the factor you want to cancel out.

•  $\frac{2x+3}{x}$  ← You can't cancel out anything here because you only have 1 term with the factor (x) above the fraction line.

•  $\frac{2x+5x^2}{x}$  ← You can cancel out an (x) from each term  
(Remember to apply exponent laws!)  
 $=(2+5x)$

### (11) h) Distribute negatives properly

•  $-4(x-3)$

$-4(x)$      $\cancel{-4(-3)}$

① Distribute -4 to each term inside the brackets.

$\cancel{-4} \quad \cancel{1}$   
 $-4x+12$

### (11) Double Brackets (FOIL)

**F**  $\downarrow$  **O**  $\rightarrow$  **I**  $\rightarrow$  **L**

$$(2x-3)(4x-1) \quad (2x)(4x) \quad (2x)(-1) \quad (-3)(4x)$$

① When you have to distribute binomials like this, use FOIL.  
**F** - First **O** - outside **I** - Inside **L** - Last

$$= 8x^2 - 2x - 12x + 3$$

$$= 8x^2 - 14x + 3$$

② Collect like terms

**DON'T ASSUME BRACKETS**

**F**  $\downarrow$   $2x - 3(4x-1)$

① Distribute  $-3$

In this example,  $-3$  is being distributed first. Then the products of that are being subtracted/added.  $\star$  ALWAYS REMEMBER BEDMAS

$$= 2x - 12x + 3$$

$$= -10x + 3$$

② Collect Like terms

$\star$  If you ever have more than 2 binomials that need to be foiled, FOIL in pairs and then continue to distribute until you can collect like terms + simplify.

### (11) Brackets and Powers

**F**  $\downarrow$   $2(x+1)^2$

$2(x+1)(x+1)$

$2(x^2 + 2x + 1)$

$= 2x^2 + 4x + 2$

① Use BEDMAS  
so, do  $(x+1)^2$  before multiplying 2 into brackets

② Multiply in (distribute) the 2

Also, the power of 2 applies only to  $(x+1)$ !

**F**  $\downarrow$   $(y^2+6y)^3$

$y^2 \times 3 + 6^1 \times 3 y^{1 \times 3}$

$= y^6 + 6^3 y^3$

$= y^6 + 216y^3$

① Use distributive property exponent law  
 $\star$  DO NOT COMMON FACTOR BEFORE

DON'T forget to apply  $^3$  to the 6!

$\star$  YOU CAN COMMON FACTOR NOW (if you wish)